

# Recent Trends in Computational Social Choice

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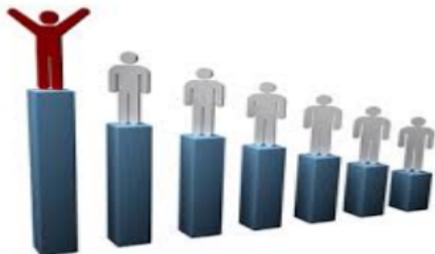
Recent Trends in Algorithms



Date: 9 February 2019

## Typical Voting Setting

- ▶ A set  $\mathcal{A}$  of  $m$  candidates
- ▶ A set  $\mathcal{V}$  of  $n$  votes
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## Example

- ▶  $\mathcal{A} = \{a, b, c\}$
- ▶ Votes
  - ✓ Vote 1:  $a > b > c$
  - ✓ Vote 2:  $c > b > a$
  - ✓ Vote 3:  $a > c > b$

**Plurality rule:** winner is candidate with most top positions

**Plurality winner:** a

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- For  $\mathcal{D} = \{\mathcal{L}(\mathcal{A})\}$ : query complexity  $\Theta(nm \log m)$

## Preference Elicitation cont.

**Single peaked domain:**  $\mathcal{O}(mn) + \mathcal{O}(m \log m)$ <sup>1</sup>

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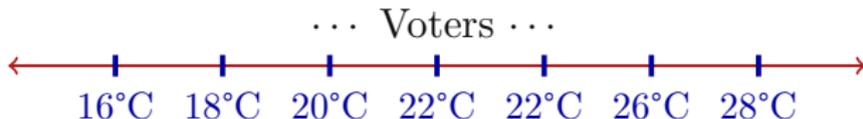
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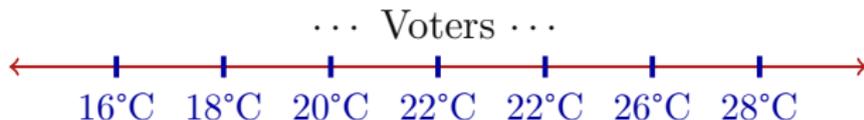
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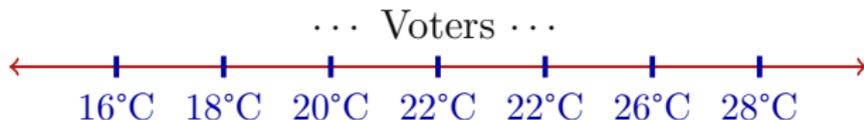
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- ▶ Random access:  $\Theta(m^2 \log n)$ <sup>2</sup>
- ▶ Sequential access:  $\mathcal{O}(mn + m^3 \log m), \Omega(mn + m^2)$

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# Preference Elicitation – open problems

## 2-Dimensional Euclidean domain:

- ▶ Alternatives  $\mathcal{A}$  are points in  $\mathbb{R}^2$  and rankings  $R_i, i \in [n]$  correspond to points  $p_i \in \mathbb{R}^2, i \in [n]$ .
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What is query complexity for 2-dimensional Euclidean domain?

## Preference Elicitation – open problems

### Single Crossing Domain on Median Graphs:

- ▶ *median graph*: for any three vertices  $u, v, w$  and for any 3 shortest paths between pairs of them  $p_{u,v}$  between  $u$  and  $v$ ,  $p_{v,w}$  between  $v$  and  $w$ , and  $p_{w,u}$  between  $w$  and  $u$ , there is exactly one vertex common to 3 paths. Ex: tree, hypercube.

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- ▶ *single crossing property*: given a median graph on some multiset  $\{R_i \in \mathcal{L}(\mathcal{A}) : i \in [n]\}$  of rankings, for every pair  $i \neq j$ , the sequence of rankings in the shortest path between  $R_i$  and  $R_j$  is single crossing.

What is query complexity of single crossing domain on median graphs?

# Winner Prediction

$r$ : any voting rule

Given an oracle which gives uniform votes of  $n$  voters over  $m$  alternatives, predict the winner under voting rule  $r$  with error probability at most  $\delta$ .

**Goal:** minimize number of samples drawn

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For  $\mathcal{A} = \{\mathbf{a}, \mathbf{b}\}$ ,  $\lfloor n/2 \rfloor - 1$  votes of type  $\mathbf{a} > \mathbf{b}$ , and  $\lceil n/2 \rceil + 1$  votes of type  $\mathbf{b} > \mathbf{a}$ , sample complexity is  $\Omega(n \ln 1/\delta)$ .

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**Margin of victory:** minimum number of votes need to modify to change the winner.

Assume: margin of victory if  $\epsilon n$ .

## Winner Prediction cont.

Plurality rule: sample complexity is  $\Theta\left(\frac{1}{\epsilon^2} \log \frac{1}{\delta}\right)$  (folklore!)

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What about other voting rules?

Voting rule	Sample complexity
Borda: $s(\mathbf{a}) = \sum_{\mathbf{b} \neq \mathbf{a}} \mathbf{N}(\mathbf{a} > \mathbf{b})$	$\Theta\left(\frac{1}{\epsilon^2} \log \frac{\log m}{\delta}\right)$
Maximin: $s(\mathbf{a}) = \min_{\mathbf{b} \neq \mathbf{a}} \mathbf{N}(\mathbf{a} > \mathbf{b})$	$\Theta\left(\frac{1}{\epsilon^2} \log \frac{\log m}{\delta}\right)$
Copeland: $s(\mathbf{a}) =  \{\mathbf{b} \neq \mathbf{a} : \mathbf{N}(\mathbf{a} > \mathbf{b}) > \frac{n}{2}\} $	$\mathcal{O}\left(\frac{1}{\epsilon^2} \log^3 \frac{\log m}{\delta}\right)$ $\Omega\left(\frac{1}{\epsilon^2} \log \frac{\log m}{\delta}\right)$

## Winner Prediction Future Directions

- ▶ What is sample complexity for winner prediction for specific domains, for example, single peaked, single crossing, and single crossing on median graphs?

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- ▶ What is sample complexity for winner prediction for specific domains, for example, single peaked, single crossing, and single crossing on median graphs?
- ▶ What is the sample complexity for committee selection rules like Chamberlin–Courant or Monroe.

## Liquid Democracy

- ▶ *If you are not sure whom you should vote, then you can delegate your friend!*<sup>34</sup>

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<sup>3</sup>J.C. Miller, “A program for direct and proxy voting in the legislative process,” *Public Choice*, 1969.

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## Liquid Democracy

- ▶ *If you are not sure whom you should vote, then you can delegate your friend!*<sup>3</sup><sup>4</sup>
- ▶ Delegations are transitive.

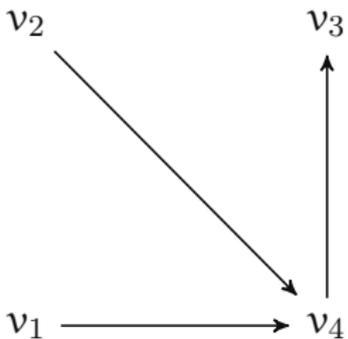


Figure 1: Delegation graph

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## Pitfalls of Liquid Democracy: Super voter

*Voting power can be concentrated in one super voter which may be undesirable even if he/she is competent.*

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*Voting power can be concentrated in one super voter which may be undesirable even if he/she is competent.*

- ▶ Natural solution: put cap on the maximum weight of a voter.
- ▶ Can lead to delegation outside system thereby reducing transparency!
- ▶ Ask voters to provide multiple delegations whom they trust and let system decide the rest.<sup>5</sup>

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# Resolving Delegation Graph

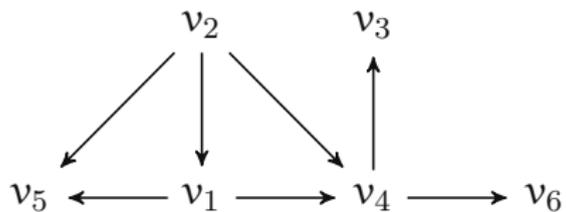


Figure 2: Input graph

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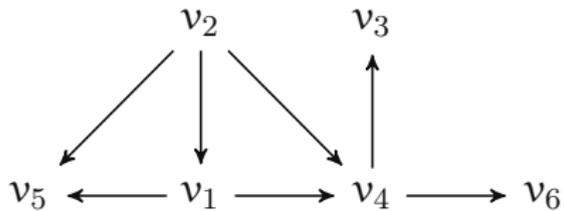


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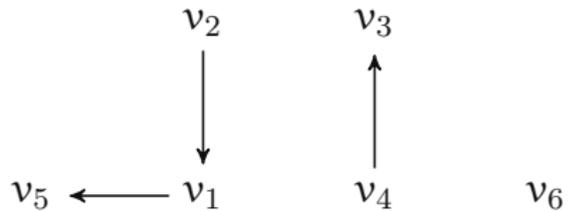


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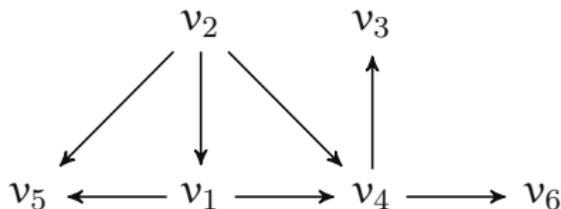


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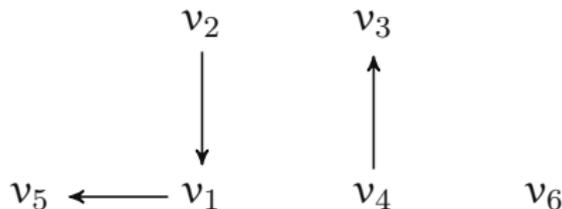


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Given a directed graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$  with sink nodes  $\mathcal{S}[\mathcal{G}]$ , find a spanning subgraph  $\mathcal{H} \subseteq \mathcal{G}$  such that  $\mathcal{S}[\mathcal{H}] \subseteq \mathcal{S}[\mathcal{G}]$  which minimizes the weight (number of nodes that can reach it) of any node.

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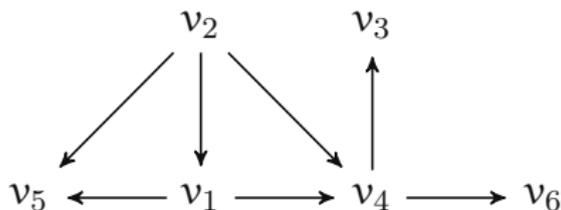


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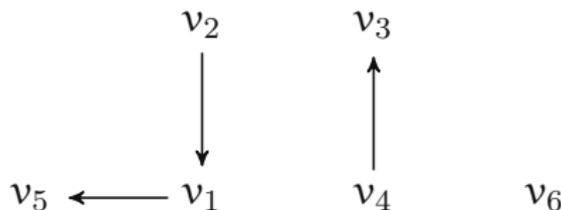


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Gölg et al. present  $1 + \lg n$  approximation and show  $\frac{1}{2} \lg n$  inapproximability assuming  $\mathbf{P} \neq \mathbf{NP}$  by reducing to the problem of minimizing maximum confluent flow.

# Restricting Voter Power is Recommended for Efficiency

## Reason too

- ▶ Assume there are only 2 choices ( $\mathcal{A} = \{0, 1\}$ ) with 0 being ground truth.
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- ▶ Gain: given a delegation mechanism, its gain is the probability that 0 wins minus 0 wins under direct voting.
- ▶ Positive Gain (PG): A mechanism is said to have PG property if its gain is positive for all sufficiently large instances.
- ▶ Do Not Harm (DNH): A mechanism is said to have DNH property if its gain is non-negative for all sufficiently large graphs

# Restricting Voter Power is Recommended for Efficiency

## Reason too cont.

- ▶ No local delegation mechanism has DNH property!<sup>6</sup>

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# Restricting Voter Power is Recommended for Efficiency

## Reason too cont.

- ▶ No local delegation mechanism has DNH property!<sup>6</sup>
- ▶ There exists a non-local mechanism which satisfies PG property and the main idea is to provide cap on the weight of any voter.

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## Participatory Budgeting

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- ▶ *Core*: An allocation  $x$  is called a core if, for every subset  $S \subseteq [n]$ , there does not exist any allocation  $y$  such that  $\sum_{i \in S} y_i \leq \frac{|S|}{n}B$  and  $U_i(y) > U_i(x)$  for every  $i \in S$ .

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- ▶ Core captures fairness notion in this context and an allocation in the core can be computed in polynomial time for a class of utility functions. <sup>7</sup>

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- ▶ Ranking by value.
- ▶ Ranking by value for money.
- ▶ For a threshold  $t$ , a feasible subset of projects which ensures an utility of at least  $t$ .

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Distortion: fraction of welfare (sum of utilities) loss due to lack of information.

Elicitation method	Distortion
Any method	$\leq m$
Knapsack vote	$\Omega(m)$
Ranking by value	$\mathcal{O}(\sqrt{m} \log m)$
Ranking by value for money	
Deterministic threshold	$\Omega(\sqrt{m})$
Randomized threshold	$\mathcal{O}(\log^2 m), \Omega\left(\frac{\log m}{\log \log m}\right)$

*What is the optimal elicitation method?*

# Distortion of Voting Rules

## *Implicit Utilitarian Voting Model*

Although votes are rankings over alternatives, every voter  $i$  has an underlying utility function  $u_i : \mathcal{A} \rightarrow [0, 1]$ ,  $\sum_{a \in \mathcal{A}} u_i(a) = 1$ .

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*Distortion of a voting rule:* what fraction of welfare ( $\sum_{i=1}^n u_i(w)$  if  $w$  wins) it achieves in the worst case compared to optimal.<sup>8</sup>

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Distortion of any randomized voting rule is  $\Omega(\sqrt{m})$ . The distortion of harmonic scoring rule ( $i$ -th ranked alternatives receives a score of  $1/i$ ) is  $\mathcal{O}(\sqrt{m \log m})$ .<sup>9</sup>

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Distortion of optimal social welfare function is  $\tilde{\Theta}(\sqrt{m})$ .<sup>10</sup>

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- ▶ Metric distortion of any rule is at least 3.<sup>11</sup>

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*Implicit Utilitarian Model. Voters and Alternatives are embedded in a metric space.*

- ▶ Metric distortion of any rule is at least 3.<sup>11</sup>
- ▶ Metric distortion of plurality and Borda are at least  $2m - 1$ , of veto and k-approval are at least  $2n - 1$ .

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# Very Hard Voting Problems

*Some natural problems in voting are  $\Sigma_2^P$ -complete and  $\Theta_2^P$ -complete.*

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Constructive Control by Deleting Alternatives (CCDA): Given a set of votes over a set of alternative and an alternative  $c$ , compute if it possible to delete at most  $k$  candidates such that  $c$  wins in the resulting election.

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# Thank You!



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