Fixed parameter tractable algorithms for corridor guarding problems

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Outline

Geometric Covering problems

- Motivated by the applications in VLSI design, and motion planning, geometric covering problems have been studied extensively.
- One has to cover geometric objects (e.g., points, lines, disks, squares or rectangles) with other geometric objects, satisfying some optimization requirements.

Motivation

- Applications in VLSI
  - Minimize the length of the wire used
  - Reduce the number of links(bends) in a path connecting two points in the board
- Most of the covering problems are NP-hard even in rectilinear domains(lines/line-segments parallel to x-axis or y-axis)[1]

[1] Jianxin Wang, Jinyi Yao, Qilong Feng, and Jianer Chen.Improved fpt algorithms for rectilinear k-links spanning path.In International Conference on Theory and Applications of Models of Computation, Springer,2012

Corridor Guarding problems

- Minimum corridor guarding problems (CMST/CTSP)
- Minimum link CTSP
- Minimum corridor connection problems

Minimum corridor guarding problems [3]

- Input: Connected orthogonal arrangement of line-segments
  Output: An optimal tree/closed walk, such that if a guard moves through the tree/closed walk, all the line-segments are visited[2] by the guard.
- If the guarding walk is a tree/closed walk, then the problem is referred to as Corridor-MST/Corridor-TSP(CMST/CTSP)
- Decision version of CMST/CTSP is proved to be NP-Complete.

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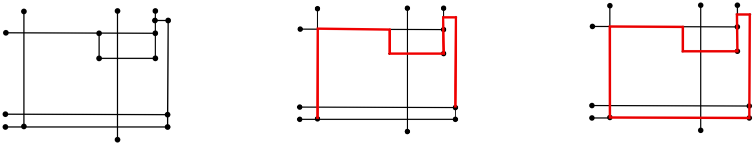
[2] a line-segment l is said to be visited by a tree/walk, if any of the vertices in the tree/walk is incident to one of the endpoints or intersection points created by l with other line-segments

[3] Ning Xu.Complexity of minimum corridor guarding problems.Information Processing Letters, 2012.

Minimum corridor guarding problems



(a) (b) (c)

Figure: (a) represents input instance of CMST and CTSP. Red lines in (b) and (c) represent the tree and closed walk respectively

Minimum link CTSP

- Given an orthogonal connected arrangement L of line-segments, find a minimum link-distance closed walk visiting all the line-segments.
- Link-distance is the number of links or turns in a path/walk.

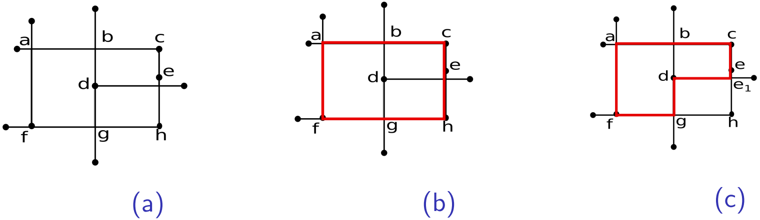


Figure: Input and Output Instances of MLC.
(a) The input arrangement of line-segments.
(b) closed walk in (a) with link-distance four(ac, ch, hf, and fa are the links)
(c) closed walk in (a) with six link-distance (ac, ce_1, e_1d, dg, gf and fa are the links) respectively.

Minimum corridor connection problems [4]

- Given a rectilinear polygon partitioned into rectilinear components or rooms, MCC asks for a minimum length tree along the edges of the partitions, such that every room is incident to at least one vertex of the tree.
- Decision version of the problem is shown to be NP-complete.

(a) (b)

Figure: Input and Output instances of MCC. (a) Rectilinear polygon partitioned into rooms. In (b) the red lines represent a minimal tree visiting all rooms

[4] Hans L Bodlaender et al.On the minimum corridor connection problem and other generalized geometric problems.Computational Geometry, 42(9), 2009.

Parameterized Complexity [5]

- A framework for solving NP-hard problems by measuring their time in terms of one or more parameters, in addition to the input size.
- A problem with input instance of size n, and with a non-negative integer parameter k, is *fixed-parameter tractable*(FPT), if it can be solved by an algorithm that runs in $O(f(k).n^c)$-time, where f is a computable function depending only on k, and c is a constant independent of k.

[5] Rolf Niedermeier.Invitation to fixed-parameter algorithms.2006

k-CMST/k-CTSP(k-Corridor-MST/k-Corridor-TSP)

Input: A connected arrangement of line-segments (corridors) $L = \{L_1, L_2, \ldots, L_n\}$, and an integer k
Parameter: k
Output: A minimum length tree/closed walk on at most k vertices, along the edges of the corridor, such that all the line-segments are visited.

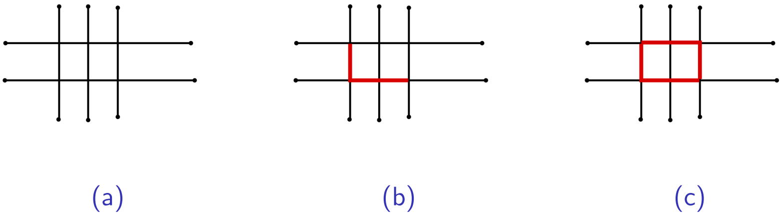
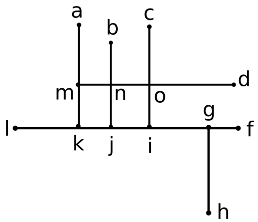


Figure: Red lines in (b) shows tree with k=4 and Red lines in (c) shows closed walk with $k = 6$ for input instance (a)

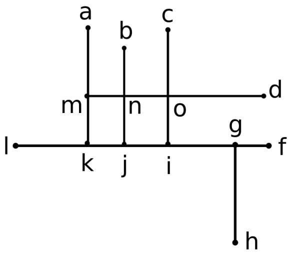
An FPT algorithm for k-CMST/k-CTSP

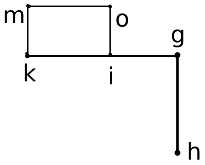
Input : Orthogonal Arrangement of line-segments



- Segment Vertices $V_s = \{ a, b, c \ldots o\}$ and Segment Edges $E_s = \{ am, bn, co, od, gf, lk, mn, no, kj, ji, mk, nj, oi, gh, ig\}$
- Isolated segment edges $E_{is} = \{ am, bn, co, od, gf, lk\}$
- Segment bounding rectangle: Rectangle formed by the set of topmost and bottommost horizontal line-segments , and leftmost and rightmost vertical line-segments when two or more horizontal(vertical) line-segments is intersected by three or more vertical(horizontal) line-segments. ($[mo, oi, ik, km]$ in the figure).

An FPT algorithm for k-CMST/k-CTSP

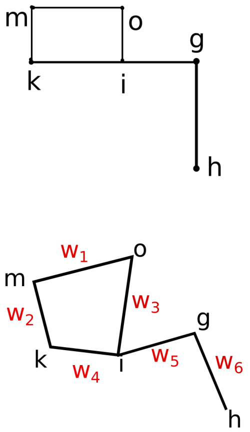




- Preprocess the input instance
  - Remove isolated-segment edges if any.
  - Remove those line segments which have both their end-points in the boundary of a segment-bounding rectangle, if any.

Parameter k is decreased by the number of line-segments removed. The updated parameter is referred to as l.

An FPT algorithm for k-CMST/k-CTSP



- Transform the preprocessed instance to graph instance G_{ls}.
  - The segment vertices and edges of the preprocessed instance is transformed into vertices and edges of the graph G_{ls}.
  - Length of the segment-edges are assigned as the weights of the corresponding edges in the graph.
- Find l-Tree cover and l-Tour cover of the graph instance

l-Tree cover/l-Tour cover (Weighted connected vertex cover)

- Input : A graph $G = (V, E, w)$ where $w : E \rightarrow \mathbb{R}^+$, an integer $l \geq 0$. Parameter : l, Number of vertices in the output tree/closed walk
  Output: A minimal Tree/closed walk $T = (V', E')$ of G with $V' \subseteq V$ and $E' \subseteq E$, $|V'| \leq l$ and V' is a vertex cover for G.
- Both l-Tree Cover and l-Tour Cover were shown to be FPT.

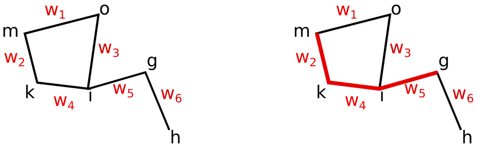


Figure: Red lines in (b) shows tree-cover with k=4 for graph in (a).

FPT result of k-CMST/k-CTSP

Lemma

l-Tree Cover and l-Tour Cover can be solved in $O((2l)^l)$ and $O((4l)^l)$-time, respectively.[a]

[a] Jiong Guo, Rolf Niedermeier, and Sebastian Wernicke. Parameterized complexity of generalized vertex cover problems. In Workshop on Algorithms and Data Structures, pages 3648. Springer, 2005.

Lemma

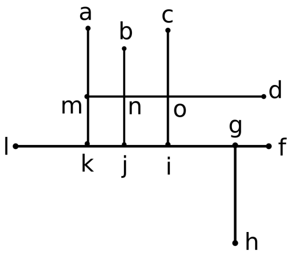
k-CMST/k-CTSP on an input instance (L', l) is an YES-instance iff l-Tree Cover/l-Tour Cover in its corresponding G_{ls} has an YES-instance.

Theorem

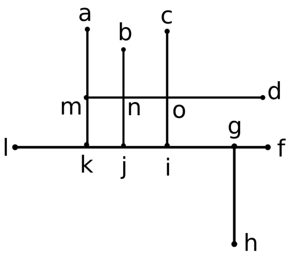
k-CMST and k-CTSP on an arrangement L is FPT with a run-time of $O^*(2k^k)$ and $O^*(4k^k)$ respectively.

An improved FPT algorithm for k-CMST/k-CTSP

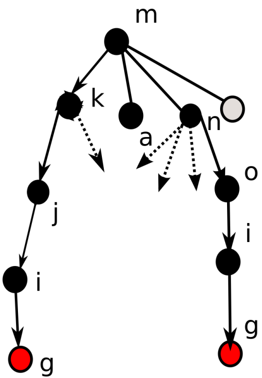
- Consider the geometric instance.
- Uses a search tree which starts with a segment-vertex with segment-degree ≥ 2.
- Each node has 4 branches, and each branch selects one segment edge.
- Branching is performed until all the line-segments in the arrangement are visited, S is a tree/closed walk and $k \geq 0$.



An improved FPT algorithm for k-CMST/k-CTSP



(a)



(b)

Figure: m is the start vertex. $m - k - j - i - g$ and $m - n - o - i - g$ are two trees with $k = 5$ vertices

An improved FPT algorithm for k-CMST/k-CTSP

- Initially, if we select a vertex which is not part of the tree/closed walk, the branching algorithm may return a NO, even when the input is a YES instance.

Lemma

If there is line-segment l in L intersected by more than k line-segments, then the instance (L, k) is a NO instance for k-CMST. If l is intersected by more than $k/2$ line-segments, then the instance is a NO instance for k-CTSP.

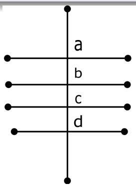


Figure: For $k < 4$ k-CMST returns a NO, and for $k < 8$ k-CTSP returns a NO

An improved FPT algorithm for k-CMST/k-CTSP

Corollary

The maximum intersections possible for a line-segment l in a YES instance of k-CMST is k, and k/2 for k-CTSP.

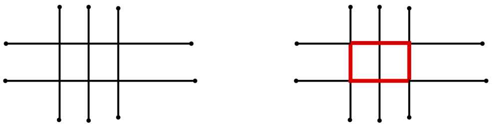
- The algorithm is invoked for a maximum of k times for k-CMST and $k/2$ times for k-CTSP (Maximum number of intersections is k and $k/2$ respectively).
- Running time : O*($k.4^k$)

Theorem

There is an $O^*(k.4^k)$-time algorithm for k-CMST and $O^*((k/2).4^k)$-time algorithm for k-CTSP. Consequently, these problems are FPT.

b-MLC (b-Minimum link Corridor-TSP)

Input: A connected arrangement of line-segments (corridors) $L = \{L_1, L_2, \ldots, L_n\}$ with bounded number of intersections m for every line-segment in L & an integer b
Parameter: b
Output: A minimum length closed walk on at most b link-distance along the edges of the corridor, such that all the line-segments are visited.



(a) (b)

Figure: Red lines in (c) shows closed walk with $b = 4$ for input instance (a)

Hardness result of b-MLC

Theorem

b-MLC is NP-complete.

Candidate problem: Point covering rectilinear tour of b links or b link point-tour.
Input: A set of n points in a plane
Question: Is there a rectilinear tour of at most b link-distance which covers all the points?

- b-link point tour is proven to be NP-Complete[6].

[6] Jianxin Wang, Jinyi Yao, Qilong Feng, and Jianer Chen.Improved fpt algorithms for rectilinear k-links spanning path.In International Conference on Theory and Applications of Models of Computation, pages 560571. Springer,2012

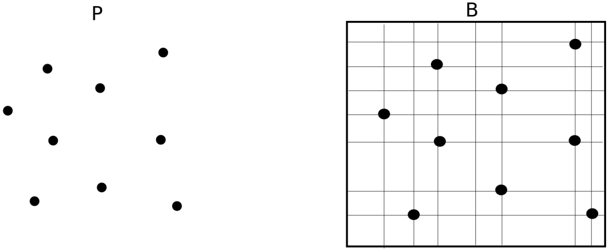


Figure: Example of reduction from point covering by a b-link tour to b-MLC.

- Enclose the points in a rectangular bounding box and build an orthogonal line arrangement of the points.
- The endpoints in the line-segments of b-MLC is either one of the original n points, or the intersection points made by the lines with the bounding box.

Hardness result of b-MLC

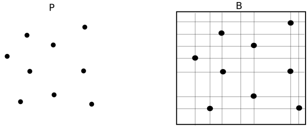


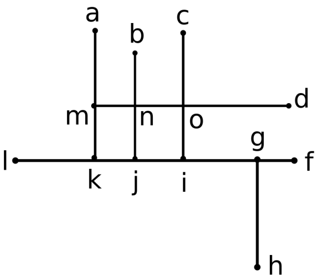
Figure: Example of reduction from point covering by a b-link tour to b-MLC.

- Every point in the input of point covering corresponds to four line-segments in b-MLC.
- It is obvious from the construction, that each of the line-segments share one of its endpoints with at least one of the n points.
- So, if there is a b-link tour connecting the n points, then there is a closed walk visiting all $4n$ line-segments with at most b link-distance.
- The decision version of the problem is in NP, the verifying algorithms checks if a sequence of line-segments forms a closed walk, visits all the line-segments, and has at most b link-distance.
- b-MLC is NP-Complete.

An FPT algorithm for b-MLC

- Uses a search tree
- Each node has 4(m+1) branches where m is the bound in number of intersections in one line-segment, and each branch selects one link.
- Branching is performed until all the line-segments in the arrangement are visited, S is a closed walk and $b \geq 0$.
- Initially, if we select a vertex which is not part of the closed walk, the branching algorithm may return a NO, even when the input is a YES instance.

An FPT algorithm for b-MLC



- The maximum number of intersections for a line-segment in the figure is 4.
- Suppose we start with the vertex l, the possible links are lk, lj, li, lg, and lf .
- If we start with m, one of the solutions is $m - o - i - g$ with 3 link-distance.

An FPT algorithm for b-MLC

- The algorithm is invoked for a maximum of m times since the maximum bound on intersection is m.
- Running time : $O(m.(4(m+1))^b)$

Input: A rectilinear polygon P partitioned into $\{P_1, P_2, \ldots, P_k\}$ rectilinear components or rooms.
Parameter: k, The number of partitions or rooms.
Output: A minimum length tree along the edges of the partitions such that all k rooms are visited.[7]

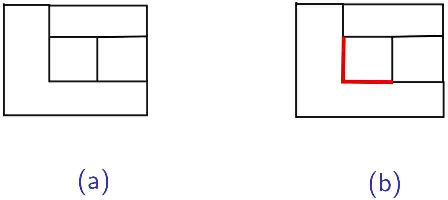
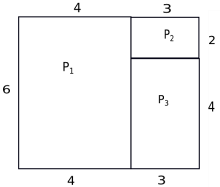


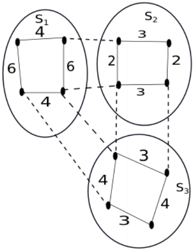
Figure: Red lines in (b) shows tree with k=4 for the input in (a).

[7] A room is said to be visited by a tree when it is incident to one of the vertices of the tree.

An FPT algorithm for k-MCC

Transform the input instance to a graph instance where the vertices are divided as k groups of terminals.





- Corresponding to each of the partitions $\{P_1, P_2, \ldots, P_k\}$ in P, group of terminals $S_1, S_2, \ldots, S_k$ in G_{pd} is created.
- Edge weights in G_{pd} are added corresponding to the length of the line-segments in the partitions of P.
- The dotted lines corresponds to the 0 weight edges which are added between vertices shared by partitions.

FPT algorithm for k-edgewt-Group Steiner tree

- In G_{pd}, find a group Steiner tree visiting all k groups.

k-edgewt-GST
Input: A connected undirected graph $G = (V, E, w)$ where $w : E \rightarrow \mathbb{R}^+$, vertex-disjoint subsets $\{S_1, S_2, \ldots, S_k\}$ where each $S_i \subseteq V \; \forall \; 1 \leq i \leq k$.
Parameter: k
Output: A minimal tree in G that includes at least one vertex from each $S_i \; \forall \; 1 \leq i \leq k$.

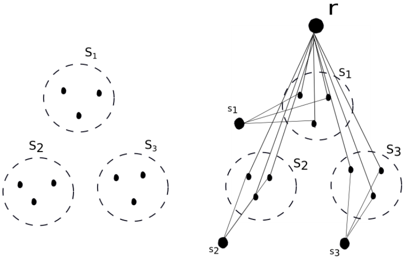
FPT algorithm for k-edgewt-Group Steiner tree

- Reduce k-edgewt-GST to k-edgewt-DST.
  k-edgewt-DST
  Input: A Directed graph $G' = (V', E', w')$ where $w^{'} : E^{'} \rightarrow \mathbb{R}^{+}$, a distinguished vertex r $\in V$, a set of terminals $S \subseteq V$ where $|S| = k$.
  Parameter: k
  Output: A minimal out-tree in G' that is rooted at r and that contains all the vertices of S.

Lemma

k-edgewt-GST has a parameter preserving reduction to k-edgewt-DST.

Transformation of instance from weighted GST in G to weighted DST



- Additional $k+1$ vertices $\{s_1, s_2, \ldots, s_k, r\}$ are included in DST instance.
- For each edge (u, v) in G, edges (u, v) and (v, u) with the same edge weights is added in D.
- An arc of length 1 is added from r to all vertices in S_i.
- An arc of length 1 is added from vertices of S_i to corresponding s_i, $\forall\ 1 \leq i \leq k$.

Transformation of instance from weighted GST in G to weighted DST

- If G contains a tree T with minimal edge-weight m that includes at least one vertex from each S_i, then this tree with the same weight m is also contained in D which can be accessed from r using one of the (r, u) arc for some $u \in V$.
- Thus we have a directed out-tree with edge-weight $(m + k + 1)$ containing r and all vertices in S.
- Also, if any one of the group S_i is omitted, then T must omit s_i.
- Thus, there is a parameterized preserving reduction from k-edgewt-GST to k-edgewt-DST.

k-MCC

Theorem

There is a $O^*(2^{O(klogk)})$-time algorithm for k-edgewt-DST.[a]

[a] Fedor V Fomin, Fabrizio Grandoni, Dieter Kratsch, Daniel Lokshtanov, and Saket Saurabh. Computing optimal steiner trees in polynomial space. Algorithmica, 2013

Theorem

k-MCC is solved in $O^*(2^{O(klogk)})$-time. Consequently, it is FPT.

Summary

Problem	Complexity Status	FPT results
k-CMST	NP-Complete [Xu12]	$O^*(2k^k)$, $O^*(k(4^k))$
k-CTSP	NP-Complete [Xu12]	$O^*(4k^k)$, $O^*((k/2)4^k)$
b-MLC	NP-Complete	$O^*(m(4(m+1))^b)$
k-MCC	NP-Complete[BFG+09]	$O^*(2^k logk)$

Future work

- To incorporate an option of visibility of rooms, in addition to the notion of visiting rooms.

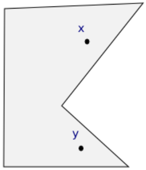


Figure: Notion of visibility: x and y is not visible to each other since the line-segment xy is not completely inside the polygon

- Another direction of work related to MLC problem is finding a tree with minimum number of links or link-diameter(maximum link-distance between any two points in the tree.)

Thank You

References I

[AGN01] Jochen Alber, Jens Gramm, and Rolf Niedermeier.
Faster exact algorithms for hard problems: a parameterized point of view.
Discrete Mathematics, 229(1-3):3–27, 2001.

[AHH93] Esther M Arkin, Magnús M Halldórsson, and Rafael Hassin.
Approximating the tree and tour covers of a graph.
Information Processing Letters, 47(6):275–282, 1993.

[BFG+09] Hans L Bodlaender, Corinne Feremans, Alexander Grigoriev, Eelko Penninkx, René Sitters, and Thomas Wolle.
On the minimum corridor connection problem and other generalized geometric problems.
Computational Geometry, 42(9):939–951, 2009.

[Bod98] Hans L Bodlaender.
A partial k-arboretum of graphs with bounded treewidth.
Theoretical computer science, 209(1-2):1–45, 1998.

[CFK+15] Marek Cygan, Fedor V Fomin, Łukasz Kowalik, Daniel Lokshtanov, Dániel Marx, Marcin Pilipczuk, Michał Pilipczuk, and Saket Saurabh.
Parameterized algorithms, volume 3.
Springer, 2015.

[CG89] Bernard Chazelle and Leonidas J Guibas.
Visibility and intersection problems in plane geometry.
Discrete & Computational Geometry, 4(6):551–581, 1989.

[CN86] Wei-Pang Chin and Simeon Ntafos.
Optimum watchman routes.
In *Proceedings of the second annual symposium on Computational geometry*, pages 24–33. ACM, 1986.

[Cyg12] Marek Cygan.
Deterministic parameterized connected vertex cover.
In *Scandinavian Workshop on Algorithm Theory*, pages 95–106. Springer, 2012.

[DBVKOS97] Mark De Berg, Marc Van Kreveld, Mark Overmars, and Otfried Schwarzkopf.
Computational geometry.
In *Computational geometry*, pages 1–17. Springer, 1997.

[DELM03] Moshe Dror, Alon Efrat, Anna Lubiw, and Joseph SB Mitchell.
Touring a sequence of polygons.
In *Proceedings of the thirty-fifth annual ACM symposium on Theory of computing*, pages 473–482. ACM, 2003.

[Deo17] Narsingh Deo.
Graph theory with applications to engineering and computer science.
Courier Dover Publications, 2017.

[DF13] Rodney G Downey and Michael R Fellows.
Fundamentals of parameterized complexity, volume 4.
Springer, 2013.

[DO00] Erik D Demaine and Joseph ORourke.
Open problems from cccg99.
In *Proc. 12th Canad. Conf. Comput. Geom*, pages 269–272, 2000.

[DT12] Adrian Dumitrescu and Csaba D Tóth.
Watchman tours for polygons with holes.
Computational Geometry, 45(7):326–333, 2012.

[DVV04] CW Duin, A Volgenant, and Stefan Voß.
Solving group steiner problems as steiner problems.
European Journal of Operational Research, 154(1):323–329, 2004.

[DW71] Stuart E Dreyfus and Robert A Wagner.
The steiner problem in graphs.
Networks, 1(3):195–207, 1971.

References III

[ECHS10a] Vladimir Estivill-Castro, Apichat Heednacram, and Francis Suraweera.
Np-completeness and fpt results for rectilinear covering problems.
J. UCS, 16(5):622–652, 2010.

[ECHS10b] Vladimir Estivill-Castro, Apichat Heednacram, and Francis Suraweera.
The rectilinear k-bends tsp.
In *International Computing and Combinatorics Conference*, pages 264–277. Springer, 2010.

[FG06] Jörg Flum and Martin Grohe.
Parameterized complexity theory.
Springer Science & Business Media, 2006.

[FGK+13] Fedor V Fomin, Fabrizio Grandoni, Dieter Kratsch, Daniel Lokshtanov, and Saket Saurabh.
Computing optimal steiner trees in polynomial space.
Algorithmica, 65(3):584–604, 2013.

[FLL03] Corinne Feremans, Martine Labbé, and Gilbert Laporte.
Generalized network design problems.
European Journal of Operational Research, 148(1):1–13, 2003.

[Gan99] Joseph L Ganley.
Computing optimal rectilinear steiner trees: A survey and experimental evaluation.
Discrete Applied Mathematics, 90(1-3):161–171, 1999.

[GGJ76] Michael R Garey, Ronald L Graham, and David S Johnson.
Some np-complete geometric problems.
In *Proceedings of the eighth annual ACM symposium on Theory of computing*, pages 10–22. ACM, 1976.

[GJ02] Michael R Garey and David S Johnson.
Computers and intractability, volume 29.
wh freeman New York, 2002.

References IV

[GNW05] Jiong Guo, Rolf Niedermeier, and Sebastian Wernicke.
Parameterized complexity of generalized vertex cover problems.
In *Workshop on Algorithms and Data Structures*, pages 36–48. Springer, 2005.

[Hee10] Apichat Heednacram.
The NP-hardness of covering points with lines, paths and tours and their tractability with FPT-algorithms.
Griffith University, 2010.

[HK14] Mathias Hauptmann and Marek Karpinski.
A compendium on steiner tree problems (cit. on p. 25).
2014.

[HRW92] Frank K Hwang, Dana S Richards, and Pawel Winter.
The Steiner tree problem, volume 53.
Elsevier, 1992.

[Jia12] Minghui Jiang.
On covering points with minimum turns.
In *Frontiers in Algorithmics and Algorithmic Aspects in Information and Management*, pages 58–69. Springer, 2012.

[KKPS04] Jochen Könemann, Goran Konjevod, Ojas Parekh, and Amitabh Sinha.
Improved approximations for tour and tree covers.
Algorithmica, 38(3):441–449, 2004.

[LYW94] DT Lee, Chung-Do Yang, and CK Wong.
On bends and distances of paths among obstacles in two-layer interconnection model.
IEEE Transactions on Computers, 43(6):711–724, 1994.

[MPR+12] Neeldhara Misra, Geevarghese Philip, Venkatesh Raman, Saket Saurabh, and Somnath Sikdar.
Fpt algorithms for connected feedback vertex set.
Journal of Combinatorial Optimization, 24(2):131–146, 2012.

References V

[MRR08] Daniel Mölle, Stefan Richter, and Peter Rossmanith.
Enumerate and expand: Improved algorithms for connected vertex cover and tree cover.
Theory of Computing Systems, 43(2):234–253, 2008.

[Ned09] Jesper Nederlof.
Fast polynomial-space algorithms using möbius inversion: Improving on steiner tree and related problems.
In *International Colloquium on Automata, Languages, and Programming*, pages 713–725. Springer, 2009.

[Nie06] Rolf Niedermeier.
Invitation to fixed-parameter algorithms.
2006.

[o'R98] Joseph o'Rourke.
Computational geometry in C.
Cambridge university press, 1998.

[PS12] Franco P Preparata and Michael I Shamos.
Computational geometry: an introduction.
Springer Science & Business Media, 2012.

[W+01] Douglas Brent West et al.
Introduction to graph theory, volume 2.
Prentice hall Upper Saddle River, 2001.

[Wag06] David Phillip Wagner.
Path planning algorithms under the link-distance metric.
2006.

[WPN92] Chin Wei-Pang and Simeon Ntafos.
The zookeeper route problem.
Information Sciences, 63(3):245–259, 1992.

References VI

[WTY$^+$14] Jianxin Wang, Peiqiang Tan, Jinyi Yao, Qilong Feng, and Jianer Chen.
On the minimum link-length rectilinear spanning path problem: complexity and algorithms.
IEEE Transactions on Computers, 63(12):3092–3100, 2014.

[WYFC12] Jianxin Wang, Jinyi Yao, Qilong Feng, and Jianer Chen.
Improved fpt algorithms for rectilinear k-links spanning path.
In *International Conference on Theory and Applications of Models of Computation*, pages 560–571. Springer, 2012.

[Xu12] Ning Xu.
Complexity of minimum corridor guarding problems.
Information Processing Letters, 112(17-18):691–696, 2012.