Simultaneous Consecutive Ones Submatrix and Editing Problems: Classical Complexity & Fixed-Parameter Tractable Results

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Joint work with
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Recent Trends in Algorithms
National Institute of Science Education and Research

Outline

Simultaneous Consecutive Ones Problem [1]

- Does a given binary matrix have the **simultaneous consecutive ones property (*SC*1*P*)** ?

$$\begin{array}{c} \\ r_1 \\ r_2 \\ r_3 \\ r_4 \end{array}\begin{pmatrix} c_1 & c_2 & c_3 & c_4 & c_5 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

[1] Alan Tucker. A structure theorem for the consecutive 1s property. Journal of Combinatorial Theory, Series B, 12(2):153162, 1972)

Simultaneous Consecutive Ones Property

- Permute the rows and columns so that the ones appear consecutively in every column and every row.

$$\begin{array}{c} \\ r_1 \\ r_2 \\ r_3 \\ r_4 \end{array} \begin{pmatrix} c_1 & c_2 & c_3 & c_4 & c_5 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

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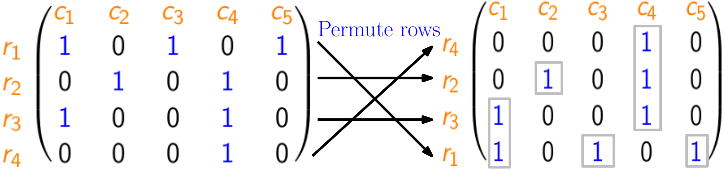

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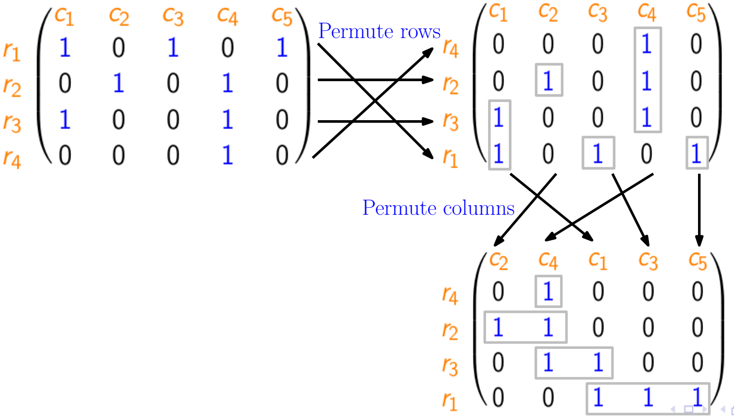

A matrix having the *SC1P*

Simultaneous Consecutive Ones Property

- Not all binary matrices have the $SC1P$.

[2] K. S. Booth, G. S. Lueker, Testing for the consecutive ones property, interval graphs, and graph planarity using pq-tree algorithms, Journal of Computer and System Sciences 13 (3) (1976) 335379.

Simultaneous Consecutive Ones Property

- Not all binary matrices have the *SC1P*.

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

A matrix not having the *SC1P*

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A matrix not having the $SC1P$

- Testing $SC1P$ - linear time [2]

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Characterization of $SC1P$

A binary matrix M has the $SC1P$ if and only if no submatrix of M is a member of the *configuration* of $M_{I_k} (k \geq 1)$, M_{2_1}, M_{2_2}, M_{3_1}, M_{3_2}, M_{3_3} or their transposes [a].

[a] Alan Tucker. A structure theorem for the consecutive 1s property. Journal of Combinatorial Theory, Series B, 12(2):153162, 1972)

$$\begin{bmatrix} 1 & 1 & 0 & . & . & . & 0 \\ 0 & 1 & 1 & 0 & . & . & 0 \\ 0 & 0 & 1 & 1 & 0 & . & 0 \\ . & . & . & . & . & . & . \\ 0 & . & . & . & 0 & 1 & 1 \\ 1 & 0 & . & . & . & 0 & 1 \end{bmatrix}$$

M_{I_k}, $k \geq 1$ ($k+2$ **rows and** $k+2$ **columns**)

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

M_{2_1}

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

M_{2_2}

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

M_{3_1}

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

M_{3_2}

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

M_{3_3}

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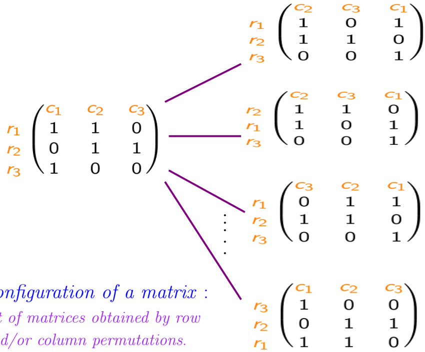

Configuration of a matrix :
set of matrices obtained by row and/or column permutations.

Problems based on SC1P

- Simultaneous Consecutive Ones Submatrix ($SC1S$) problems

[3] Marcus Oswald and Gerhard Reinelt. The simultaneous consecutive ones problem. Theoretical Computer Science, 410(21-23):19861992, 2009.

Problems based on SC1P

- Simultaneous Consecutive Ones Submatrix (*SC1S*) problems
  - SC1S-row deletion.

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Problems based on SC1P

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- Simultaneous Consecutive Ones Editing ($SC1E$) problems

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  - SC1P-0-flipping

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  - SC1P-0-flipping
  - SC1P-1-flipping [3]
  - SC1P-01-flipping

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Applications

- In Bioinformatics

[4] R. Knig, G. Schramm, M. Oswald, H. Seitz, S. Sager, M. Zapatka, G. Reinelt, R. Eils, Discovering functional gene expression patterns in the metabolic network of escherichia coli with wavelets transforms, BMC bioinformatics 7 (1) (2006) 119.

Applications

- In Bioinformatics
  - To discover functionally meaningful patterns from gene expression data[4].

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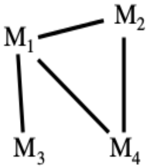
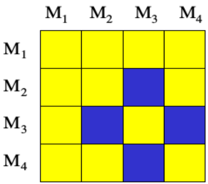


Metabolic network of gene-expression data



Adjacency matrix of metabolites

- represents 1-entries
- represents 0-entries

- Gene expression data mapped onto metabolic network.
- An adjacency matrix of metabolites was created.
- Consecutive ones clustering method used to obtain network clusters.

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Biconvex graphs & $SC1P$

Biconvex Graphs

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Biconvex Graphs

A bipartite graph $G = (V_1, V_2, E)$ is *biconvex* if both V_1 and V_2 can be ordered so that for every vertex v in $V_1 \cup V_2$, neighbors of v occur consecutively in the ordering.

Biconvex graphs & $SC1P$

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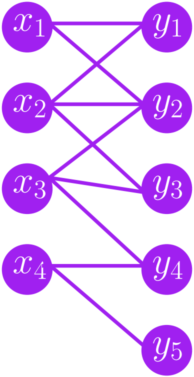

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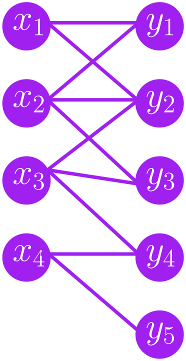

Ordering of V_1 : x_1, x_2, x_3, x_4

Ordering of V_2 : y_1, y_2, y_3, y_4, y_5

Biconvex graphs & *SC1P*

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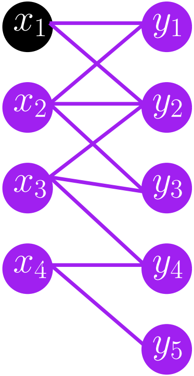

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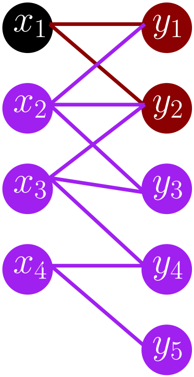

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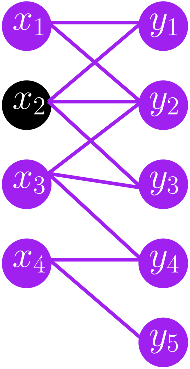

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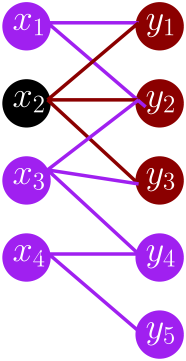

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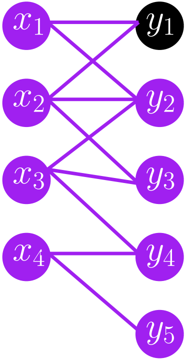

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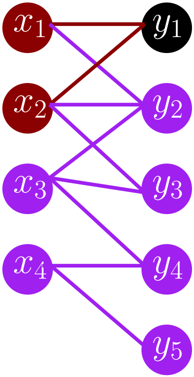

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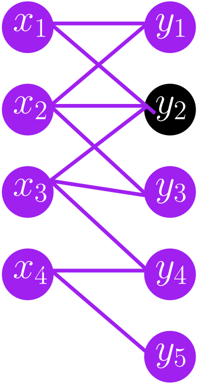

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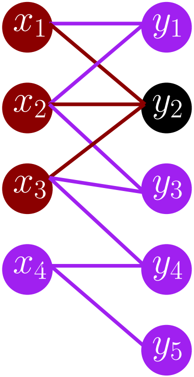

Ordering of $V_1 : x_1, x_2, x_3, x_4$

Ordering of $V_2 : y_1, y_2, y_3, y_4, y_5$

Characterization of Biconvex graphs

A bipartite graph $G = (V_1, V_2, E)$ is biconvex if and only if its half adjacency matrix has the $SC1P$ [a] .

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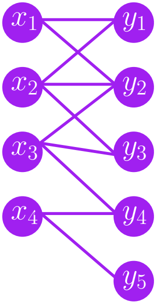

Bipartite Graph

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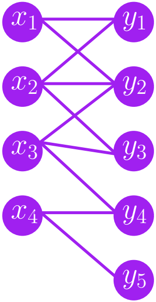
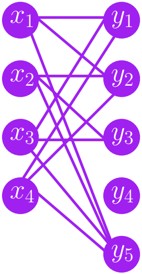


Bipartite Graph

$$\begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{pmatrix} y_1 & y_2 & y_3 & y_4 & y_5 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

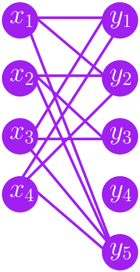
Half-adjacency matrix



A bipartite graph $G = (V_1, V_2, E)$ *(not biconvex)*
$V_1 = \{x_1, x_2, x_3, x_4\}$ and $V_2 = \{y_1, y_2, y_3, y_4, y_5\}$

	y_1	y_2	y_3	y_4	y_5
x_1	1	1	0	0	1
x_2	0	1	1	0	1
x_3	1	0	1	0	1
x_4	1	1	0	0	1

Half-adjacency matrix of G, $M(G)$

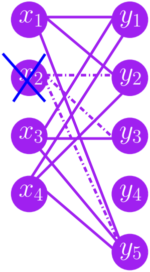


A bipartite graph G (not biconvex)

	y_1	y_2	y_3	y_4	y_5
x_1	1	1	0	0	1
~~x_2~~	~~0~~	~~1~~	~~1~~	~~0~~	~~1~~
x_3	1	0	1	0	1
x_4	1	1	0	0	1

Half-adjacency matrix of G, $M(G)$

Biconvex graphs : Constrained Vertex Deletion



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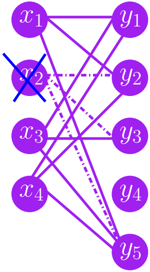

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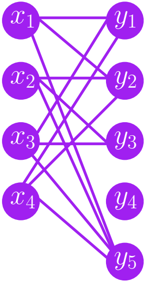
Half-adjacency matrix of G, M(G)

SC1S-row deletion ⇔ *Problem of finding a minimum number of vertices to be deleted from V_1, so that the resultant graph is biconvex.*



A bipartite graph G (not biconvex)

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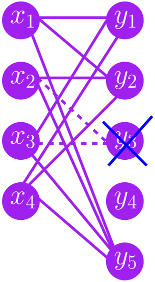
Half-adjacency matrix of G, $M(G)$

Biconvex graphs : Constrained Vertex Deletion



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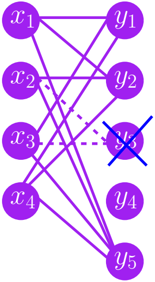
Half-adjacency matrix of G, M(G)



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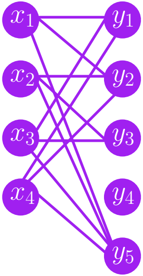
Half-adjacency matrix of G, M(G)

SC1S-column deletion $\Leftrightarrow$ *Problem of finding a minimum number of vertices to be deleted from V_2, so that the resultant graph is biconvex.*

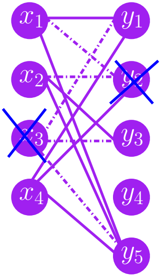


A bipartite graph G (not biconvex)

	y_1	y_2	y_3	y_4	y_5
x_1	1	1	0	0	1
x_2	0	1	1	0	1
x_3	1	0	1	0	1
x_4	1	1	0	0	1

Half-adjacency matrix of G, $M(G)$

Biconvex graphs : Vertex Deletion

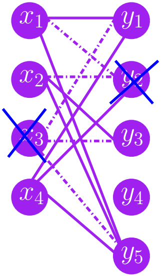


A bipartite graph G (not biconvex)

	y_1	y_2	y_3	y_4	y_5
x_1	1	1	0	0	1
x_2	0	1	1	0	1
x_3	1	0	1	0	1
x_4	1	1	0	0	1

Half-adjacency matrix of G, M(G)

Biconvex graphs : Vertex Deletion



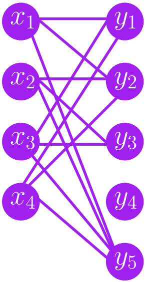
A bipartite graph G (not biconvex)

	y_1	y_2	y_3	y_4	y_5
x_1	1	1	0	0	1
x_2	0	1	1	0	1
x_3	1	0	1	0	1
x_4	1	1	0	0	1

Half-adjacency matrix of G, $M(G)$

SC1S-row & column deletion $\Leftrightarrow$ *Problem of finding a minimum number of vertices to be deleted from $V_1 \cup V_2$, so that the resultant graph is biconvex.*

Biconvex graphs : Completion

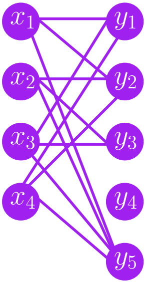


A bipartite graph G (not biconvex)

$$\begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{pmatrix} y_1 & y_2 & y_3 & y_4 & y_5 \\ 1 & 1 & \boxed{0} & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Half-adjacency matrix of G, $M(G)$

Biconvex graphs : Completion

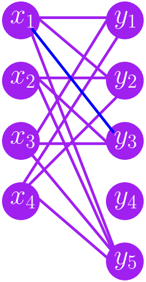


A bipartite graph G (not biconvex)

	y_1	y_2	y_3	y_4	y_5
x_1	1	1	1	0	1
x_2	0	1	1	0	1
x_3	1	0	1	0	1
x_4	1	1	0	0	1

Half-adjacency matrix of G, $M(G)$

Biconvex graphs : Completion

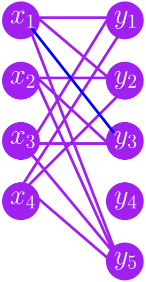


A bipartite graph G (not biconvex)

$$\begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{pmatrix} y_1 & y_2 & y_3 & y_4 & y_5 \\ 1 & 1 & \boxed{1} & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Half-adjacency matrix of G, M(G)

Biconvex graphs : Completion



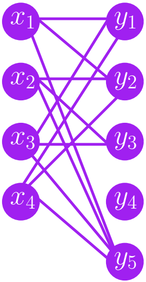
A bipartite graph G (not biconvex)

	y_1	y_2	y_3	y_4	y_5
x_1	1	1	1	0	1
x_2	0	1	1	0	1
x_3	1	0	1	0	1
x_4	1	1	0	0	1

Half-adjacency matrix of G, M(G)

SC1P-0-Flipping $\Leftrightarrow$ *Problem of finding a minimum number of non-edges to be added to G, so that the resultant graph is biconvex.*

Biconvex graphs : Edge Deletion

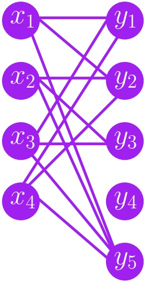


A bipartite graph G (not biconvex)

	y_1	y_2	y_3	y_4	y_5
x_1	1	1	0	0	1
x_2	0	1	1	0	1
x_3	1	0	1	0	1
x_4	1	1	0	0	1

Half-adjacency matrix of G, $M(G)$

Biconvex graphs : Edge Deletion

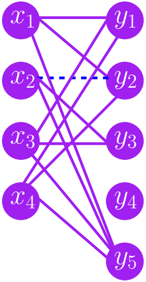


A bipartite graph G (not biconvex)

$$
\begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array}
\begin{pmatrix}
y_1 & y_2 & y_3 & y_4 & y_5 \\
1 & 1 & 0 & 0 & 1 \\
0 & \boxed{0} & 1 & 0 & 1 \\
1 & 0 & 1 & 0 & 1 \\
1 & 1 & 0 & 0 & 1
\end{pmatrix}
$$

Half-adjacency matrix of G, M(G)

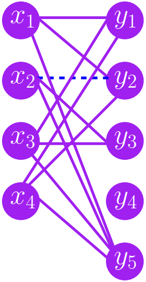
Biconvex graphs : Edge Deletion



A bipartite graph G (not biconvex)

	y_1	y_2	y_3	y_4	y_5
x_1	1	1	0	0	1
x_2	0	0	1	0	1
x_3	1	0	1	0	1
x_4	1	1	0	0	1

Half-adjacency matrix of G, M(G)



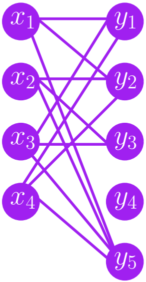
A bipartite graph G (not biconvex)

	y_1	y_2	y_3	y_4	y_5
x_1	1	1	0	0	1
x_2	0	**0**	1	0	1
x_3	1	0	1	0	1
x_4	1	1	0	0	1

Half-adjacency matrix of G, $M(G)$

SC1P-1-Flipping $\Leftrightarrow$ *Problem of finding a minimum number of edges to be deleted from G, so that the resultant graph is biconvex.*

Biconvex graphs : Edge Modification

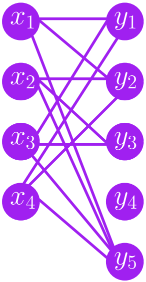


A bipartite graph G (not biconvex)

$$
\begin{array}{c|ccccc}
 & y_1 & y_2 & y_3 & y_4 & y_5 \\
\hline
x_1 & 1 & 1 & 0 & 0 & 1 \\
x_2 & 0 & 1 & \boxed{1} & 0 & 1 \\
x_3 & 1 & 0 & 1 & \boxed{0} & 1 \\
x_4 & 1 & 1 & 0 & 0 & 1
\end{array}
$$

Half-adjacency matrix of G, M(G)

Biconvex graphs : Edge Modification

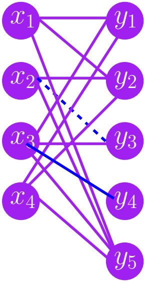


A bipartite graph G (not biconvex)

$$
\begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array}
\begin{pmatrix}
y_1 & y_2 & y_3 & y_4 & y_5 \\
1 & 1 & 0 & 0 & 1 \\
0 & 1 & \boxed{0} & 0 & 1 \\
1 & 0 & 1 & \boxed{1} & 1 \\
1 & 1 & 0 & 0 & 1
\end{pmatrix}
$$

Half-adjacency matrix of G, $M(G)$

Biconvex graphs : Edge Modification

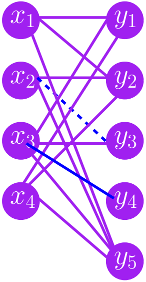


A bipartite graph G (not biconvex)

	y_1	y_2	y_3	y_4	y_5
x_1	1	1	0	0	1
x_2	0	1	0	0	1
x_3	1	0	1	1	1
x_4	1	1	0	0	1

Half-adjacency matrix of G, $M(G)$

Biconvex graphs : Edge Modification



A bipartite graph G (not biconvex)

$$\begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{pmatrix} y_1 & y_2 & y_3 & y_4 & y_5 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & \boxed{0} & 0 & 1 \\ 1 & 0 & 1 & \boxed{1} & 1 \\ 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Half-adjacency matrix of G, $M(G)$

SC1P-01-Flipping $\Leftrightarrow$ *Problem of finding a minimum number of edges to be added/deleted to/from G, so that the resultant graph is biconvex.*

Classical Complexity Results

- Decision version of $SC1S$ and $SC1E$ problems are NP-complete.

NP-completeness of k-$SC1S$-R

Hamiltonian-Path $\leq_p$ k-$SC1S$-R

NP-completeness of k-$SC1S$-R

Hamiltonian-Path $\leq_p$ k-$SC1S$-R

Transformation

Graph $G = (V, E)$, where $|V| = n$ and $|E| = m$.
Edge vertex incidence matrix $M(G)_{m \times n}$, where $k = |m| - |n| + 1$.

NP-completeness of k-$SC1S$-R

Hamiltonian-Path $\leq_p$ k-$SC1S$-R

Transformation

Graph $G = (V, E)$, where $|V| = n$ and $|E| = m$.
Edge vertex incidence matrix $M(G)_{m \times n}$, where $k = |m| - |n| + 1$.

- G has a Hamiltonian path $\Leftrightarrow$

NP-completeness of k-$SC1S$-R

Hamiltonian-Path $\leq_p$ k-$SC1S$-R

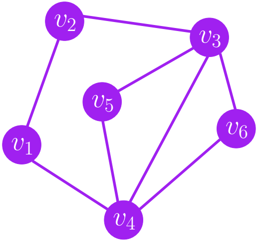
Transformation

Graph $G = (V, E)$, where $|V| = n$ and $|E| = m$.
Edge vertex incidence matrix $M(G)_{m \times n}$, where $k = |m| - |n| + 1$.

- G has a Hamiltonian path $\Leftrightarrow$ there exists a set of rows of size k in $M(G)$ whose deletion results in a matrix $M^{'}(G)$, that satisfy the $SC1P$.

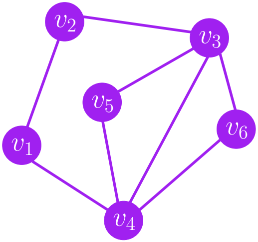
NP-completeness of *k*-*SC1S*-*R*

Forward direction:



G

Forward direction:



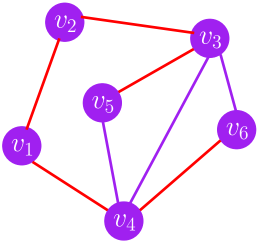
G

	v_1	v_2	v_3	v_4	v_5	v_6
$\{v_1, v_4\}$	1	0	0	1	0	0
$\{v_1, v_2\}$	1	1	0	0	0	0
$\{v_2, v_3\}$	0	1	1	0	0	0
$\{v_3, v_4\}$	0	0	1	1	0	0
$\{v_4, v_5\}$	0	0	0	1	1	0
$\{v_3, v_5\}$	0	0	1	0	1	0
$\{v_3, v_6\}$	0	0	1	0	0	1
$\{v_4, v_6\}$	0	0	0	1	0	1

$M(G)$

NP-completeness of k-$SC1S$-R

Forward direction:



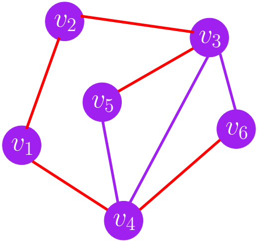
G

	v_1	v_2	v_3	v_4	v_5	v_6
$\{v_1, v_4\}$	1	0	0	1	0	0
$\{v_1, v_2\}$	1	1	0	0	0	0
$\{v_2, v_3\}$	0	1	1	0	0	0
$\{v_3, v_4\}$	0	0	1	1	0	0
$\{v_4, v_5\}$	0	0	0	1	1	0
$\{v_3, v_5\}$	0	0	1	0	1	0
$\{v_3, v_6\}$	0	0	1	0	0	1
$\{v_4, v_6\}$	0	0	0	1	0	1

$M(G)$

NP-completeness of k-$SC1S$-R

Forward direction:



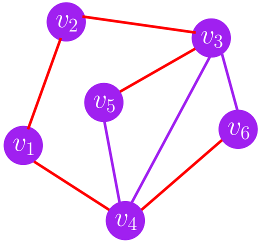
G

	v_1	v_2	v_3	v_4	v_5	v_6
$\{v_1, v_4\}$	1	0	0	1	0	0
$\{v_1, v_2\}$	1	1	0	0	0	0
$\{v_2, v_3\}$	0	1	1	0	0	0
$\{v_3, v_5\}$	0	0	1	0	1	0
$\{v_4, v_6\}$	0	0	0	1	0	1

$M'(G)$

NP-completeness of k-$SC1S$-R

Forward direction:



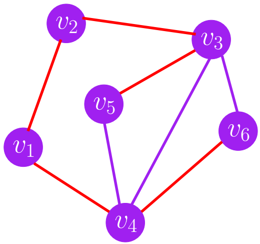
G

	v_1	v_2	v_3	v_4	v_5	v_6
$\{v_1, v_4\}$	1	0	0	1	0	0
$\{v_1, v_2\}$	1	1	0	0	0	0
$\{v_2, v_3\}$	0	1	1	0	0	0
$\{v_3, v_5\}$	0	0	1	0	1	0
$\{v_4, v_6\}$	0	0	0	1	0	1

$M'(G)$

Rearrange the rows and columns of $M'(G)$ with respect to the sequence of edges and vertices respectively in the hamiltonian path.

Forward direction:



G

	v_1	v_2	v_3	v_4	v_5	v_6
$\{v_1, v_4\}$	1	0	0	1	0	0
$\{v_1, v_2\}$	1	1	0	0	0	0
$\{v_2, v_3\}$	0	1	1	0	0	0
$\{v_3, v_5\}$	0	0	1	0	1	0
$\{v_4, v_6\}$	0	0	0	1	0	1

$M'(G)$

Rearrange the rows and columns of $M'(G)$ with respect to the sequence of edges and vertices respectively in the hamiltonian path.

	v_6	v_4	v_1	v_2	v_3	v_5
$\{v_6, v_4\}$	1	1	0	0	0	0
$\{v_1, v_4\}$	0	1	1	0	0	0
$\{v_1, v_2\}$	0	0	1	1	0	0
$\{v_2, v_3\}$	0	0	0	1	1	0
$\{v_3, v_5\}$	0	0	0	0	1	1

$M'(G)$

NP-completeness of *k*-*SC*1*S*-*R*

Reverse direction:

$$\begin{array}{c} \\ \{v_6, v_4\} \\ \{v_3, v_6\} \\ \{v_1, v_4\} \\ \{v_1, v_2\} \\ \{v_2, v_3\} \\ \{v_4, v_5\} \\ \{v_3, v_5\} \\ \{v_4, v_3\} \end{array} \begin{array}{c} \begin{array}{cccccc} v_6 & v_4 & v_1 & v_2 & v_3 & v_5 \end{array} \\ \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

M

NP-completeness of k-$SC1S$-R

Reverse direction:

$$M = \begin{array}{c} \\ \{v_6, v_4\} \\ \{v_3, v_6\} \\ \{v_1, v_4\} \\ \{v_1, v_2\} \\ \{v_2, v_3\} \\ \{v_4, v_5\} \\ \{v_3, v_5\} \\ \{v_4, v_3\} \end{array} \begin{array}{c} \begin{array}{cccccc} v_6 & v_4 & v_1 & v_2 & v_3 & v_5 \end{array} \\ \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

$$M' = \begin{array}{c} \\ \{v_6, v_4\} \\ \{v_1, v_4\} \\ \{v_1, v_2\} \\ \{v_2, v_3\} \\ \{v_3, v_5\} \end{array} \begin{array}{c} \begin{array}{cccccc} v_6 & v_4 & v_1 & v_2 & v_3 & v_5 \end{array} \\ \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \end{array}$$

NP-completeness of k-$SC1S$-R

Reverse direction:

	v_6	v_4	v_1	v_2	v_3	v_5
$\{v_6, v_4\}$	1	1	0	0	0	0
$\{v_3, v_6\}$	1	0	0	0	1	0
$\{v_1, v_4\}$	0	1	1	0	0	0
$\{v_1, v_2\}$	0	0	1	1	0	0
$\{v_2, v_3\}$	0	0	0	1	1	0
$\{v_4, v_5\}$	0	1	0	0	0	1
$\{v_3, v_5\}$	0	0	0	0	1	1
$\{v_4, v_3\}$	0	1	0	0	1	0

M

	v_6	v_4	v_1	v_2	v_3	v_5
$\{v_6, v_4\}$	1	1	0	0	0	0
$\{v_1, v_4\}$	0	1	1	0	0	0
$\{v_1, v_2\}$	0	0	1	1	0	0
$\{v_2, v_3\}$	0	0	0	1	1	0
$\{v_3, v_5\}$	0	0	0	0	1	1

M'

Consider G', the subgraph obtained from M' by considering M' as an edge- vertex incidence matrix.

NP-completeness of k-$SC1S$-R

Reverse direction:

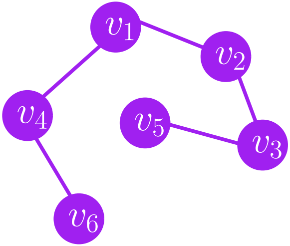
	v_6	v_4	v_1	v_2	v_3	v_5
$\{v_6, v_4\}$	1	1	0	0	0	0
$\{v_3, v_6\}$	1	0	0	0	1	0
$\{v_1, v_4\}$	0	1	1	0	0	0
$\{v_1, v_2\}$	0	0	1	1	0	0
$\{v_2, v_3\}$	0	0	0	1	1	0
$\{v_4, v_5\}$	0	1	0	0	0	1
$\{v_3, v_5\}$	0	0	0	0	1	1
$\{v_4, v_3\}$	0	1	0	0	1	0

M

	v_6	v_4	v_1	v_2	v_3	v_5
$\{v_6, v_4\}$	1	1	0	0	0	0
$\{v_1, v_4\}$	0	1	1	0	0	0
$\{v_1, v_2\}$	0	0	1	1	0	0
$\{v_2, v_3\}$	0	0	0	1	1	0
$\{v_3, v_5\}$	0	0	0	0	1	1

$M^{'}$

Consider G', the subgraph obtained from M' by considering M' as an edge- vertex incidence matrix.



$G^{'}$

NP-completeness of k-$SC1S$-C

- k-$SC1S$-C problem : transpose of k-$SC1S$-R problem.

NP-completeness of k-$SC1S$-C

- consider M as the vertex-edge incidence matrix.
- k as the number of columns to be deleted.

NP-completeness of *k-SC1S-RC*

Biconvex Vertex Deletion $\leq_p$ *k-SC1S-RC*

BICONVEX VERTEX DELETION

Instance: A bipartite graph $G = (V_1, V_2, E)$ and an integer $k \geq 0$.
Question: Does there exist a set of vertices of size at most k in G, whose deletion results in a biconvex graph?

[5] Mihalis Yannakakis. Node-deletion problems on bipartite graphs. SIAM Journal on Computing, 10(2):310327, 1981.

NP-completeness of *k-SC1S-RC*

Biconvex Vertex Deletion $\leq_p$ *k-SC1S-RC*

BICONVEX VERTEX DELETION

Instance: A bipartite graph $G = (V_1, V_2, E)$ and an integer $k \geq 0$.
Question: Does there exist a set of vertices of size at most k in G, whose deletion results in a biconvex graph?

Biconvex Vertex Deletion problem is NP-complete [5].

[5] Mihalis Yannakakis. Node-deletion problems on bipartite graphs. SIAM Journal on Computing, 10(2):310327, 1981.

NP-completeness of *k-SC1S-RC*

Biconvex Vertex Deletion $\leq_p$ *k-SC1S-RC*

BICONVEX VERTEX DELETION

Instance: A bipartite graph $G = (V_1, V_2, E)$ and an integer $k \geq 0$.
Question: Does there exist a set of vertices of size at most k in G, whose deletion results in a biconvex graph?

Biconvex Vertex Deletion problem is NP-complete [5].

Transformation

A bipartite graph $G = (V_1, V_2, E)$, where $|V_1| = n_1$, $|V_2| = n_2$ and $|E| = m$.
Half-adjacency matrix $M_{G_{n_1 \times n_2}}$.

[5] Mihalis Yannakakis. Node-deletion problems on bipartite graphs. SIAM Journal on Computing, 10(2):310327, 1981.

NP-completeness of *k-SC1S-RC*

- Claim: G has a set of vertices of size at most k, whose deletion results in a biconvex graph $\Leftrightarrow$

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- The above claim follows from the characterization of biconvex graphs relating its half-adjacency matrices and the $SC1P$.

NP-completeness of k-$SC1P$-$0E$

k-Chain-Completion $\leq_p$ k-$SC1P$-$0E$

NP-completeness of k-$SC1P$-$0E$

k-Chain-Completion $\leq_p$ k-$SC1P$-$0E$

Chain Graphs

- Bipartite graph $G = (V_1, V_2, E)$ with a linear ordering of the vertices in V_1 [a].
- $N(u_1) \subseteq N(u_2) \subseteq N(u_3) \subseteq \ldots \subseteq N(u_{|V_1|})$

[a] Assaf Natanzon, Ron Shamir, and Roded Sharan. Complexity classification of some edge modification problems. Lecture notes in computer science, pages 6577, 1999.

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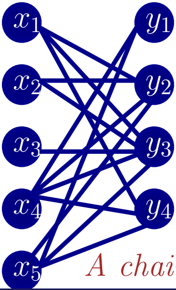


A chain graph

$$N(x_3) = \{y_3\}$$
$$N(x_2) = \{y_2, y_3\}$$
$$N(x_1) = \{y_2, y_3, y_4\}$$
$$N(x_4) = \{y_1, y_2, y_3, y_4\}$$
$$N(x_5) = \{y_1, y_2, y_3, y_4\}$$
$$N(x_3) \subseteq N(x_2) \subseteq N(x_1) \subseteq N(x_4) \subseteq N(x_5)$$

NP-completeness of k-$SC1P$-$0E$

Chain Completion

- Finding a minimum number of non-edges to be added to a given bipartite graph so that the graph becomes a chain graph.

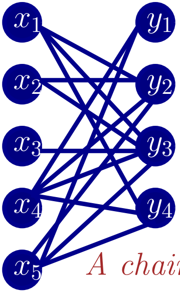
NP-completeness of k-$SC1P$-$0E$

Chain Completion

- Finding a minimum number of non-edges to be added to a given bipartite graph so that the graph becomes a chain graph.
- The decision version of the problem - k-Chain Completion is NP-complete [a].

[a] Pl Grns Drange, Markus Sortland Dregi, Daniel Lokshtanov, and Blair D Sullivan. On the threshold of intractability. In Algorithms-ESA 2015, pages 411423. Springer, 2015.

- The half adjacency matrix of a chain graph doesn't contain $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ as a submatrix and satisfies the $SC1P$.



A chain graph G

	y_1	y_2	y_3	y_4
x_3	0	0	1	0
x_2	0	1	1	0
x_1	0	1	1	1
x_4	1	1	1	1
x_5	1	1	1	1

Half-adjacency matrix of G

NP-completeness of k-$SC1P$-$0E$

Transformation

- A bipartite graph $G = (V_1, V_2, E)$, with $|V_1| = n_1$, $|V_2| = n_2$ and $|E| = m$, where $M_{G_{n_1 \times n_2}}$ is the half adjacency matrix of G.
- Matrix $M = \begin{bmatrix} J_{m,n} & M_{G_{n_1 \times n_2}} \\ 0_{m,n} & J_{m,n} \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & \mathbf{1} & \mathbf{0} & 0 \\ 1 & 1 & \mathbf{0} & \mathbf{1} & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & \underline{\mathbf{1}} & 1 & 1 & \underline{\mathbf{1}} & \underline{\mathbf{0}} & 0 \\ 1 & 1 & 1 & 1 & \underline{\mathbf{1}} & 1 & 1 & \underline{\mathbf{0}} & \underline{\mathbf{1}} & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & \underline{\mathbf{0}} & 1 & 1 & \underline{\mathbf{1}} & \underline{\mathbf{1}} & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

NP-completeness of k-$SC1P$-$0E$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$M_{I_k},\ k \geq 1: \quad \overbrace{\begin{pmatrix} 1 & 1 & 0 & \cdots & & 0 \\ 0 & 1 & 1 & 0 & \cdots & 0 \\ & & \cdots & & & \\ 0 & \cdots & & 0 & 1 & 1 \\ 1 & 0 & \cdots & & 0 & 1 \end{pmatrix}}^{k+2} \Bigg\} k+2$$

$$M_{I_1} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \qquad M_{I_2} = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

$$M_{I_3} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad M_{I_4} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

NP-completeness of k-$SC1P$-$01E$

k-Chain-Editing $\leq_p$ k-$SC1P$-$01E$

NP-completeness of k-$SC1P$-01E

k-Chain-Editing $\leq_p$ k-$SC1P$-01E

Chain Editing

- Finding a minimum number of edges to be added and removed from a given bipartite graph so that the graph becomes a chain graph.
- The decision version of the problem, k-Chain Editing is NP-complete[a].

[a] Pl Grns Drange, Markus Sortland Dregi, Daniel Lokshtanov, and Blair D Sullivan. On the threshold of intractability. In Algorithms-ESA 2015, pages 411423. Springer, 2015.

NP-completeness of k-$SC1P$-$01E$

Transformation

- A bipartite graph $G = (V_1, V_2, E)$, with $|V_1| = n_1$, $|V_2| = n_2$ and $|E| = m$, where $M_{G_{n_1 \times n_2}}$ is the half adjacency matrix of G.
- Matrix $M = \begin{bmatrix} J_{m,mn} & M_G \\ 0_{mn,mn} & J_{mn,n} \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & \mathbf{1} & \mathbf{0} & 0 \\ 1 & 1 & \mathbf{0} & \mathbf{1} & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & \cdots & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & \cdots & \underline{\mathbf{1}} & 1 & 1 & \underline{\mathbf{1}} & \underline{\mathbf{0}} & 0 \\ 1 & 1 & 1 & \cdots & \underline{\mathbf{1}} & 1 & 1 & \underline{\mathbf{0}} & \underline{\mathbf{1}} & 0 \\ 1 & 1 & 1 & \cdots & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & \cdots & \underline{\mathbf{0}} & 1 & 1 & \underline{\mathbf{1}} & \underline{\mathbf{1}} & 1 \\ 0 & 0 & 0 & \cdots & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 1 & 1 & 1 & 1 & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Parameterized Versions of $SC1S$ & $SC1E$ problems

Simultaneous Consecutive Ones Submatrix Problems

Instance:

Parameterized Versions of $SC1S$ & $SC1E$ problems

Simultaneous Consecutive Ones Submatrix Problems

Instance: $< M, d >$- An $m \times n$ binary matrix M and an integer $d \geq 0$.

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Parameterized Versions of $SC1S$ & $SC1E$ problems

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Instance: $< M, d >$- An $m \times n$ binary matrix M and an integer $d \geq 0$.
Parameter: d.
d-SC1P-1E: Does there exist a set of 1-entries of size at most d in M whose flipping results in a matrix with the $SC1P$?

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d-$SC1P$-$01E$: Does there exist a set of entries of size at most d in M whose flipping results in a matrix with the $SC1P$?

FPT algorithms for $SC1S$ and $SC1E$ problems

- Use forbidden submatrix characterization of $SC1P$.

$$\begin{bmatrix} 1 & 1 & 0 & . & . & . & 0 \\ 0 & 1 & 1 & 0 & . & . & 0 \\ 0 & 0 & 1 & 1 & 0 & . & 0 \\ . & . & . & . & . & . & . \\ 0 & . & . & . & 0 & 1 & 1 \\ 1 & 0 & . & . & . & 0 & 1 \end{bmatrix}$$

M_{I_k}, $k \geq 1$ ($k+2$ **rows and** $k+2$ **columns**)

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

M_{2_1}

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

M_{2_2}

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

M_{3_1}

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

M_{3_2}

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

M_{3_3}

A subset of the forbidden submatrices for the $SC1P$.

- fixed size forbidden submatrices : $\{M_{2_1}, M_{2_2}, M_{3_1}, M_{3_2}, M_{3_3}, M_{2_1}^T, M_{2_2}^T, M_{3_1}^T, M_{3_2}^T, M_{3_3}^T\}$.

- Use forbidden submatrix characterization of $SC1P$.

$$\begin{bmatrix} 1 & 1 & 0 & . & . & . & 0 \\ 0 & 1 & 1 & 0 & . & . & 0 \\ 0 & 0 & 1 & 1 & 0 & . & 0 \\ . & . & . & . & . & . & . \\ 0 & . & . & . & 0 & 1 & 1 \\ 1 & 0 & . & . & . & 0 & 1 \end{bmatrix}$$

M_{I_k}, $k \geq 1$ ($k+2$ **rows and** $k+2$ **columns**)

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M_{2_1}

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M_{3_2}

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- M_{I_k} and $M_{I_k}^T$ (where $k \geq 1$) : size unbounded.

FPT algorithms for $SC1S$ and $SC1E$ problems

- Use forbidden submatrix characterization of $SC1P$.
  - fixed size forbidden submatrices : $\{M_{2_1}, M_{2_2}, M_{3_1}, M_{3_2}, M_{3_3}, M_{2_1}^T, M_{2_2}^T, M_{3_1}^T, M_{3_2}^T, M_{3_3}^T\}$.
  - M_{I_k} and $M_{I_k}^T$ (where $k \geq 1$) : size unbounded.
- Given a matrix M, detection of forbidden submatrices [6]
  - Fixed size : in $O(m^6.n)$-time.

[6] Michael Dom. Recognition, Generation, and Application of Binary Matrices with the Consecutive Ones Property. Cuvillier, 2009.

FPT algorithms for $SC1S$ and $SC1E$ problems

- Use forbidden submatrix characterization of $SC1P$.
  - fixed size forbidden submatrices : $\{M_{2_1}, M_{2_2}, M_{3_1}, M_{3_2}, M_{3_3}, M_{2_1}^T, M_{2_2}^T, M_{3_1}^T, M_{3_2}^T, M_{3_3}^T\}$.
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- Given a matrix M, detection of forbidden submatrices [6]
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  - M_{I_k} and $M_{I_k}^T$ (where $k \geq 1$) : $O(m^3 n^3)$-time.

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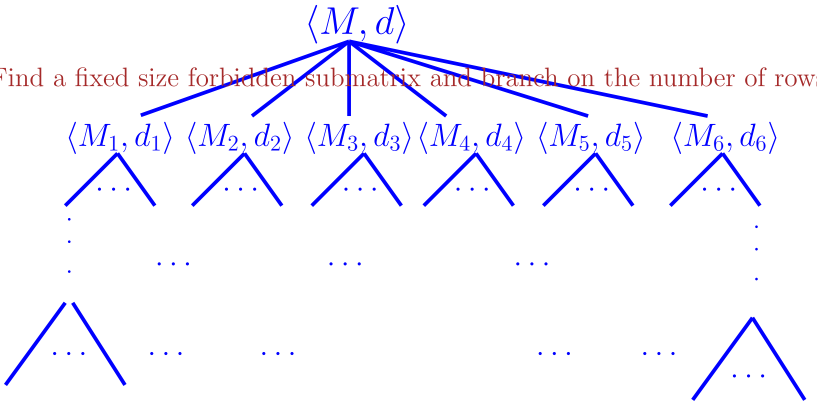
An FPT algorithm for *d-SC1S-R* problem



Apply d-COS-R[1] algorithm to each of the leaf instances to destroy M_{I_k} and $M_{I_k}^T$.

[1]N. Narayanaswamy, R. Subashini, Obtaining matrices with the consecutive ones property by row deletions, Algorithmica 71 (3) (2015) 758773.

d-COS-R subroutine runs in $O^*(8^d)$- time. (O^* notation ignores the polynomial factors and focuses on exponential part.)

An FPT algorithm for d-$SC1S$-R problem

d-COS-R

Instance: A binary matrix M and an integer $d \geq 0$.
Question:

An FPT algorithm for d-$SC1S$-R problem

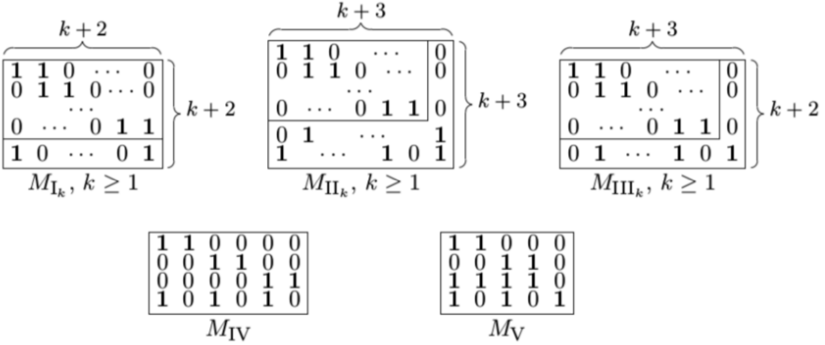
d-COS-R

Instance: A binary matrix M and an integer $d \geq 0$.
Question: Does there exist a set of rows of size at most d in M, whose deletion results in a matrix with the $C1P$ for rows?

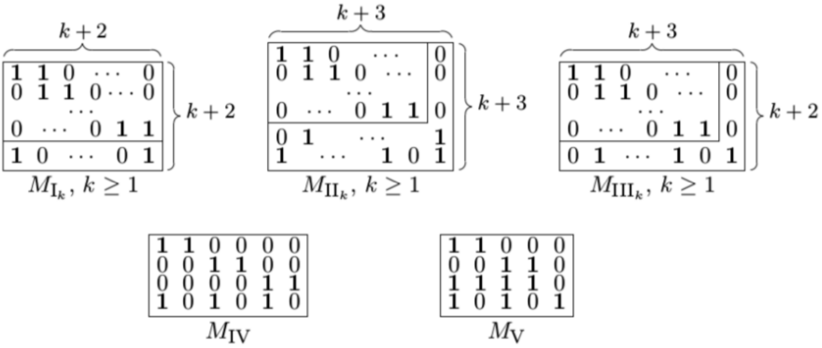
An FPT algorithm for *d-SC1S-R* problem

d-COS-R

Instance: A binary matrix M and an integer $d \geq 0$.
Question: Does there exist a set of rows of size at most d in M, whose deletion results in a matrix with the $C1P$ for rows?

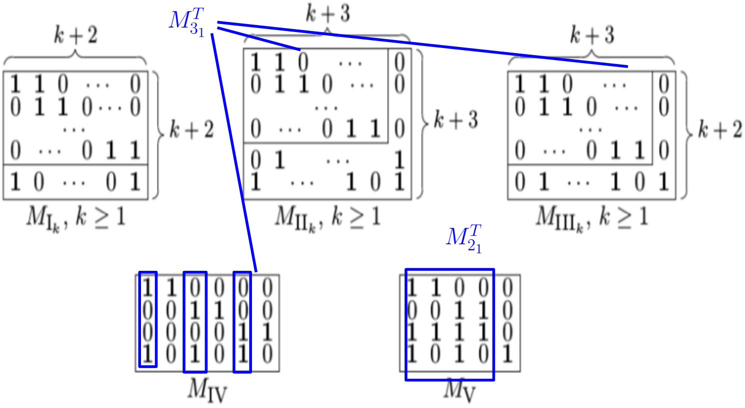


An FPT algorithm for *d-SC1S-R* problem



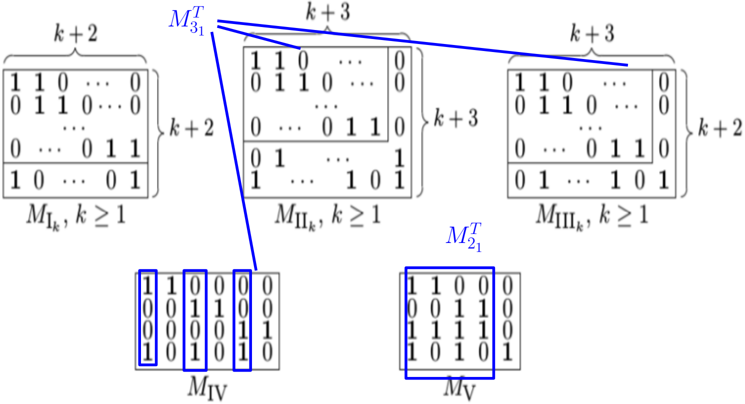
- *One of the fixed-size forbidden matrices occurs as a submatrix of every matrix in the above figure except M_{I_k}.*

An FPT algorithm for *d-SC1S-R* problem



- *One of the fixed-size forbidden matrices occurs as a submatrix of every matrix in the above figure except* M_{I_k}.

An FPT algorithm for *d-SC1S-R* problem



- *One of the the fixed-size forbidden matrices occurs as a submatrix of every matrix in the above figure except M_{I_k}.*
- *Applying d-COS-R algorithm on a leaf instance destroys only forbidden matrices of type M_{I_k}.*

An FPT algorithm for d-$SC1S$-R problem

- d_1 row deletions - destroying fixed-size forbidden matrices.
- d_2 row deletions - destroying M_{I_k} and $M_{I_k}^T$ (where $k \geq 1$)
- $d_1 + d_2 \leq d$.
- Time taken to destroy the finite size forbidden matrices is $O(6^{d_1})$
- Time taken to destroy the non-finite size forbidden matrices is $O(8^{d_2})$.
- Total run-time of the algorithm is $O^*(8^d)$.

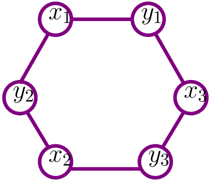
An FPT algorithm for d-$SC1P$-$0E$

Observation

The representing graph of an M_{I_k} and $M_{I_k}^T$ (where $k \geq 1$) is a chordless cycle of length $2k+4$.

	y_1	y_2	y_3
x_1	1	1	0
x_2	0	1	1
x_3	1	0	1

M_{I_1}



Representing graph $G_{M_{I_1}}$

Observation

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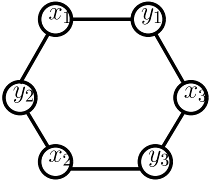

Representing graph $G_{M_{I_1}}$

- Flipping a 0-entry in M is equivalent to adding an edge in the representing graph of M.

An FPT algorithm for *d-SC1P-0E*

Observation

The representing graph of an M_{I_k} and $M_{I_k}^T$ (where $k \geq 1$) is a chordless cycle of length $2k+4$.

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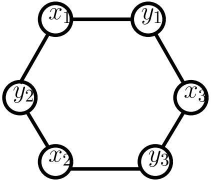
M_{I_1}



Representing graph $G_{M_{I_1}}$

- Flipping a 0-entry in M is equivalent to adding an edge in the representing graph of M.
- To destroy M_{I_k} and $M_{I_k}^T$- sufficient to destroy chordless cycles of length greater than four in the representing graph of M.

Chordal Bipartite graph

- A **chordal bipartite graph** is a bipartite graph which does not contain chordless cycles of length greater than four.

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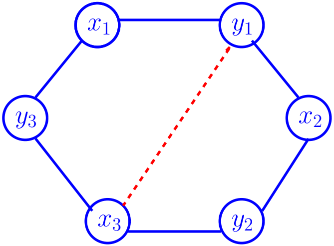
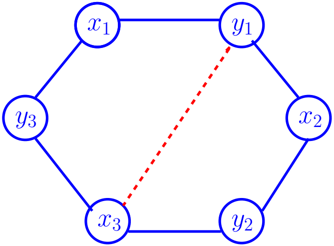


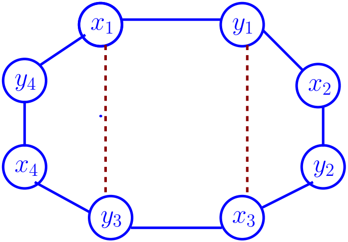
$No\ of\ edges\ to\ be\ added = 1$

Chordal Bipartite graph

- A **chordal bipartite graph** is a bipartite graph which does not contain chordless cycles of length greater than four.



$No\ of\ edges\ to\ be\ added = 1$



$No\ of\ edges\ to\ be\ added = 2$

An FPT algorithm for d-$SC1P$-$0E$

Lemma1

Let $H = (V_1, V_2, E)$ be a chordless cycle of length $2k + 4$ (where $k \geq 1$). Then, the minimum number of edges to be added to H so that H is chordal bipartite is k and the number of ways to do this is at most 6.75^{k+1}[a].

[a] H. Kaplan, R. Shamir, R. E. Tarjan, Tractability of parameterized completion problems on chordal, strongly chordal, and proper interval graphs, SIAM Journal on Computing 28 (5) (1999) 19061922.

Corollary

The minimum number of 0-flippings required to destroy an M_{I_k} or $M_{I_k}^T$, where $k \geq 1$ is k.

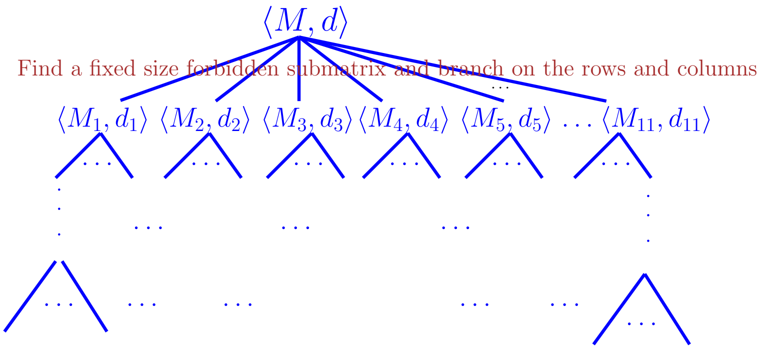
Lemma2

The total time required to destroy all M_{I_k} and $M_{I_k}^T$ in M is $O^*(6.75^d)$.

An FPT algorithm for d-$SC1P$-$0E$

- Given a binary matrix M and a nonnegative integer d,
  - If M has a forbidden matrix of type M_{I_k} and $M_{I_k}^T$ where $k > d$, immediately return NO.
  - Otherwise find a minimum size forbidden matrix in M and branch into at most 18 subcases.
- Running time : $O^*(18^d)$.

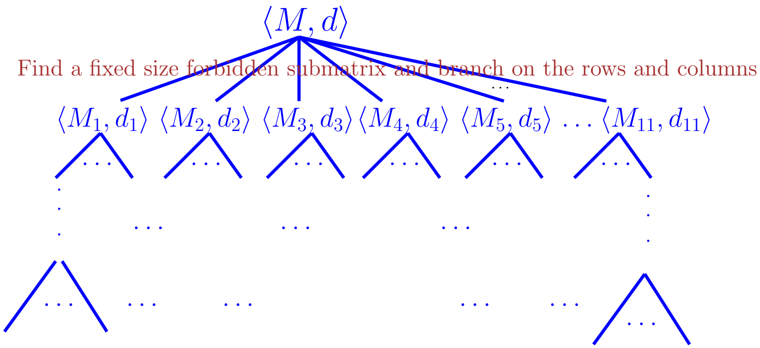
An FPT algorithm for *d-SC1S-RC* problem



- Reduce each of the leaf instance to an instance of CHORDAL VERTEX DELETION [7] problem.

[7] Y. Cao, D. Marx, Chordal editing is fixed-parameter tractable, Algorithmica 75 (1) (2016) 118137.

An FPT algorithm for *d-SC1S-RC* problem



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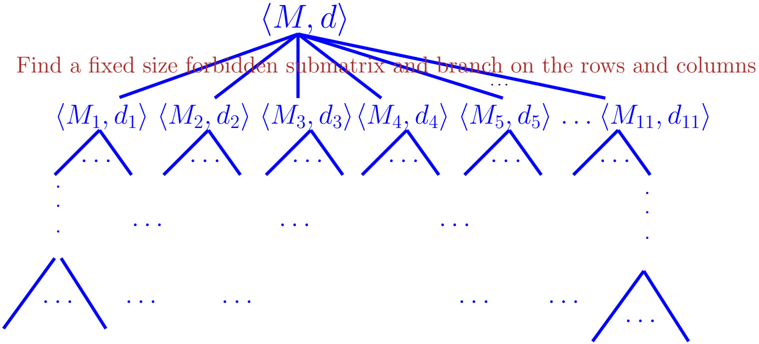
An FPT algorithm for *d-SC1S-RC* problem



- Reduce each of the leaf instance to an instance of CHORDAL VERTEX DELETION [7] problem.
  - Kill shorter chordless cycles of lenth six, eight and ten.

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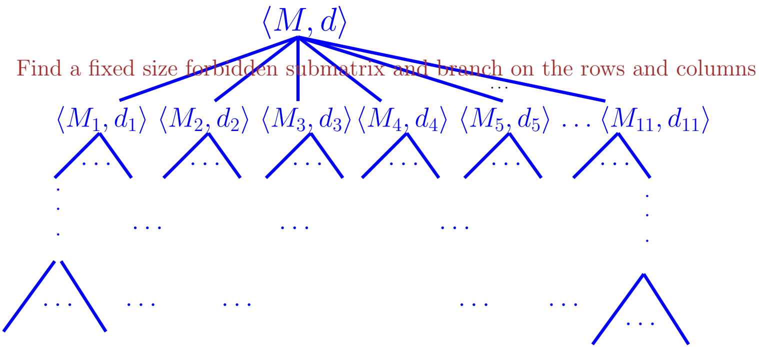
An FPT algorithm for *d-SC1S-RC* problem



- Reduce each of the leaf instance to an instance of CHORDAL VERTEX DELETION [7] problem.
  - Kill shorter chordless cycles of lenth six, eight and ten.
  - Reduce each four cycle to an edge.

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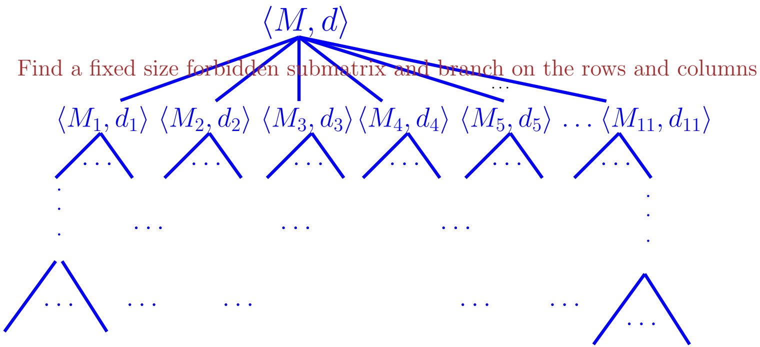
An FPT algorithm for *d-SC1S-RC* problem



- Reduce each of the leaf instance to an instance of CHORDAL VERTEX DELETION [7] problem.
  - Kill shorter chordless cycles of lenth six, eight and ten.
  - Reduce each four cycle to an edge.
  - Remove all degree 1 vertices.

[7] Y. Cao, D. Marx, Chordal editing is fixed-parameter tractable, Algorithmica 75 (1) (2016) 118137.

An FPT algorithm for *d-SC1S-RC* problem

- d_1 row deletions for destroying fixed-size forbidden matrices.
- d_2 row deletions for destroying M_{I_k} and $M_{I_k}^T$ (where $k \geq 1$)
- $d_1 + d_2 \leq d$.
- Time taken to destroy the finite size forbidden matrices is $O(11^{d_1})$
- Chordal Vertex Deletion algorithm runs in $O^*(2^{d_2 logd_2})$.
- Total run-time of the algorithm is $O^*(2^{dlogd})$.

FPT algorithm for d-$SC1S$-R on $(2,*)$[8]-matrices

- Forbidden submatrices :
  $M_{3_1}^T, M_{I_k} (k \geq 1), M_{I_k}^T (k \geq 1)$

[8] A $(2,*)$-matrix have at most two ones per column and any number of ones per row

FPT algorithm for d-$SC1S$-R on $(2,*)^8$-matrices

- Forbidden submatrices :
  $M_{3_1}^T, M_{I_k} (k \geq 1), M_{I_k}^T (k \geq 1)$
  - Destroy every submatrix of type $M_{3_1}^T$ in M.

[8] A $(2, *)$-matrix have at most two ones per column and any number of ones per row

FPT algorithm for *d-SC1S-R* on $(2,*)^8$-matrices

- Forbidden submatrices : $M_{3_1}^T, M_{I_k}(k \geq 1), M_{I_k}^T(k \geq 1)$
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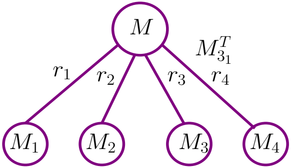

[8] A $(2,*)$-matrix have at most two ones per column and any number of ones per row

FPT algorithm for d-$SC1S$-R on $(2,*)^8$-matrices

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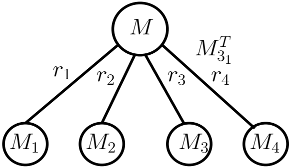

  - Preprocess the resultant matrix M to remove identical rows and columns.

[8] A $(2,*)$-matrix have at most two ones per column and any number of ones per row

FPT algorithm for d-$SC1S$-R on $(2,*)^8$-matrices

- Forbidden submatrices : $M_{3_1}^T, M_{I_k}(k \geq 1), M_{I_k}^T(k \geq 1)$
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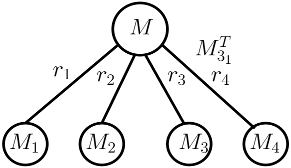

  - Preprocess the resultant matrix M to remove identical rows and columns.
  - If M still does not have $SC1P$, then remaining forbidden submatrices are pairwise disjoint.

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FPT algorithm for d-$SC1S$-R on $(2,*)^8$-matrices

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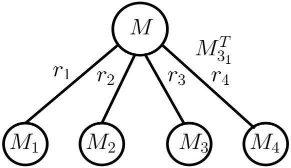

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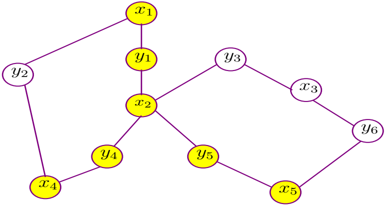

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FPT algorithm for *d-SC1S-R* on $(2,*)^8$-matrices

- Forbidden submatrices : $M_{3_1}^T, M_{I_k}(k \geq 1), M_{I_k}^T(k \geq 1)$
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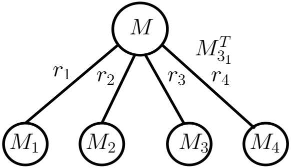

  - Preprocess the resultant matrix M to remove identical rows and columns.
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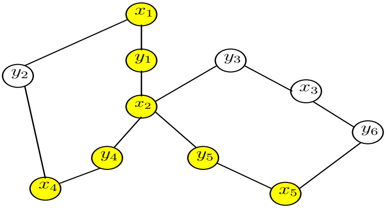

	y_1	$y2$	y_3	y_4	y_5	y_6
x_1	1	1	0	0	0	0
x_2	1	0	1	1	1	0
x_3	0	0	1	0	0	1
x_4	0	1	0	1	0	0
x_5	0	0	0	0	1	1

[8] A $(2,*)$-matrix have at most two ones per column and any number of ones per row

FPT algorithm for d-$SC1S$-R on $(2,*)$-matrices

- Uses a search tree.
- Each node : four subproblems.
- Size of search tree : $O(4^d)$.
- A submatrix isomorphic to $M_{3_1}^T$: $O(m^4 n)$-time.
- Time required for Stage 1 : $O(4^d m^4 n)$.
- M_{I_k} and $M_{I_k}^T$: $O(n^3 m^3)$ time.
- Number of M_{I_k} and $M_{I_k}^T$ in M : $O(min(m,n))$
- Size of search tree : $O(4^d)$.
- Time required for Stage 2 : $O(4^d m^3 n^3)$.
- Total time complexity : $O(4^d(m^4 n + m^3 n^3))$

d-$SC1S$-R on a $(2,*)$-matrix $M_{m \times n}$, can be solved in $O^*(4^d)$-time, where d denotes the number of rows that can be deleted. Consequently it is FPT.

Concluding Remarks

- Decision versions of $SC1S$ & $SC1E$ problems : poly-time solvable on $(2,2)$[9]-matrices.

Parameterized results			
Problem	$(2,*)$-matrix	$(*,2)$-matrix	general-matrix
d-$SC1S$-$R\backslash C$	$O^*(4^d \backslash 3^d)$	$O^*(3^d \backslash 4^d)$	$O^*(8^d)$
d-$SC1S$-RC	$O^*(7^d)$	$O^*(7^d)$	$O^*(2^{dlogd})$
d-$SC1P$-$0E$	-	-	$O^*(18^d)$
d-$SC1P$-$1E$	$O^*(6^d)$	$O^*(6^d)$	?
d-$SC1P$-$01E$	-	-	?

[9] $(2,2)$-matrix have at most two ones per row and at most two ones per column

Thank You